



# Open problems in topology

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## Abstract

This is a cumulative status report on the 1100 problems listed in the volume *Open Problems in Topology* (North-Holland, 1990), edited by J. van Mill and G.M. Reed.

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## Introduction

This is a cumulative status report on the 1100 problems listed in the volume *Open Problems in Topology* (North-Holland, 1990), edited by J. van Mill and G.M. Reed [192]. The book is out-of-print but the publisher has made it freely available online. This report is a complete revision of the seven status reports that have appeared in the journal *Topology and its Applications* [193–198,221].

This report contains a matrix (Figs. 1 and 2) of numbers indicating the status of each problem. On the matrix, a numbered box is shaded if the problem has been answered absolutely or shown to be independent of ZFC. A numbered box is half-shaded if the problem has been answered in part, for a special case, or consistently, since the volume was published. There are 199 fully shaded boxes and 76 half-shaded boxes. It is remarkable that three-quarters of the problems remain open after thirteen years. We hope that this report and the availability of the book will regenerate interest in these problems.

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### Dow's questions by A. Dow

**Problem 2.** Find necessary and sufficient conditions on a compact space  $X$  such that  $\omega \times X$  has remote points. In the notes to this problem, Dow conjectured that there is a model satisfying that if  $X$  is compact and  $\omega \times X$  has remote points then  $X$  has an open subset with countable cellularity. However, Dow [93] showed that there is a compact, nowhere c.c.c. space  $X$  such that  $\omega \times X$  has remote points.

**Problem 5.** Yes. J. Baker and K. Kunen [15] proved that if  $\kappa$  is a regular cardinal, then there is a weak  $P_{\kappa^+}$ -point in  $U(\kappa)$ , the space of uniform ultrafilters on  $\kappa$ . Problem 5 only asked for the case  $\kappa = \omega_1$ . The weak  $P_{\kappa^+}$ -point problem is still open for singular cardinals  $\kappa$ .

**Problem 8.** Is there a clopen subset of the subuniform ultrafilters of  $\omega_1$  whose closure in  $\beta\omega_1$  is its one point compactification? Yes, under PFA (S. Todorćević [274, §8]).

**Problem 9.** A. Dow and J. Vermeer [100] proved that it is consistent that the  $\sigma$ -algebra of Borel sets of the unit interval is not the quotient of any complete Boolean algebra. By Stone duality, there is a basically disconnected space of weight  $\mathfrak{c}$  that cannot be embedded into an extremally disconnected space.

**Problem 13.** Is every compact space of weight  $\omega_1$  homeomorphic to the remainder of a  $\psi$ -space? A. Dow and R. Frankiewicz [94] showed that a negative answer is consistent.

**Problem 14** (A. Błaszczyk). Is there a compact c.c.c. space of weight  $\mathfrak{c}$  whose density is not less than  $\mathfrak{c}$ ? M. Rabus and S. Shelah [227] proved that every uncountable cardinal can be the topological density of a c.c.c. Boolean algebra.

**Problem 16.** Does countable closed tightness imply countable tightness in compact spaces? I. Juhász and Z. Szentmiklóssy [143] proved that if  $\kappa$  is an uncountable regular cardinal and a compact Hausdorff space  $X$  contains a free sequence of length  $\kappa$ , then  $X$  also contains such a sequence that is convergent.

### Steprāns's problems by J. Steprāns

**Problem 19.** Yes, there is an  $\omega$ -Toronto space. An  $\alpha$ -Toronto space is a scattered space of Cantor–Bendixson rank  $\alpha$  which is homeomorphic to each of its subspaces of rank  $\alpha$ . G. Gruenhage and J. Moore [128] constructed countable  $\alpha$ -Toronto spaces for each  $\alpha \leq \omega$ . Gruenhage also constructed consistent examples of countable  $\alpha$ -Toronto spaces for each  $\alpha < \omega_1$ .

**Problem 20.** Yes, J. Steprāns constructed a homogeneous, idempotent filter on  $\omega$ .

**Problem 23.** Solved by A. Krawczyk.

**Problem 24.** Solved by A. Krawczyk.

**Problem 26.** No, S. Schuder [245] proved that  $I(2) \leq_{[0,1]} A(2)$  does not hold. The graph  $G = \{(x, x - \frac{1}{3}) : \frac{1}{3} \leq x \leq \frac{2}{3}\}$  on  $[0, 1]$  can be coloured by  $f : [0, 1] \rightarrow \{0, 1\}$ ,  $f(x) = 1$  iff  $\frac{1}{3} \leq x < \frac{2}{3}$ , but there is no  $A(2)$ -colouring  $g : [0, 1] \rightarrow A(2)$  for  $G$ .

**Problem 30.** *If every autohomeomorphism of  $\mathbb{N}^*$  is somewhere trivial, is then every autohomeomorphism trivial?* This is the same as Problem 205. S. Shelah [251, IV] proved that it is consistent that all autohomeomorphisms are trivial. S. Shelah and J. Steprāns [258] proved that it is consistent with  $\text{MA}_{\omega_1}$  that there is a nontrivial autohomeomorphism of  $\mathbb{N}^*$ , yet all autohomeomorphisms are somewhere trivial. In [259], they prove that  $\text{MA}$  does not imply that all autohomeomorphisms are somewhere trivial.

### Tall's problems by F.D. Tall

**Problem 43.** Yes, G. Gruenhage and P. Koszmider [126] constructed a consistent counterexample to the Arhangel'skiĭ–Tall problem: a locally compact, normal, metacompact space that is not paracompact.

**Problem 44.** See S. Watson's Problem 92.

**Problem 48.** *If  $\kappa$  is a singular strong limit cardinal and  $X$  is a  $< \kappa$ -collectionwise Hausdorff and normal (or countably paracompact) space of character  $< \kappa$ , is then  $X$  is  $\kappa$ -collectionwise Hausdorff?* Under SCH, N. Kemoto [156] proved this conjecture.

**Problem 49.** P. Szeptycki [270] proved that there are Easton models where first countable,  $\aleph_1$ -para-Lindelöf spaces are collectionwise Hausdorff.

**Problem 50.** T. LaBerge and A. Landver [172] proved from a supercompact cardinal that it is consistent that first countable,  $< \aleph_2$ -collectionwise Hausdorff spaces are weakly  $\leq \aleph_2$ -collectionwise Hausdorff.

**Problem 57.** Yes, there is a ZFC example of a screenable normal space that is not collectionwise normal. By a theorem of M.E. Rudin [233], it suffices to construct a screenable normal space that is not paracompact. Z. Balogh [20] constructed such a space. Balogh's example provides a positive answer to Problem 119.

**Problem 63.** *Does  $2^{\aleph_0} < 2^{\aleph_1}$  imply there is an  $S$ -space? (or an  $L$ -space?)* T. Eisworth, P. Nyikos and S. Shelah [115] proved that it is consistent with  $2^{\aleph_0} < 2^{\aleph_1}$  that there are no locally compact, first countable  $S$ -spaces.

**Problem 66.** P. Larson and F.D. Tall [173] proved that there is a c.c.c. partial order forcing that every hereditarily normal, first countable space satisfying the countable chain condition is hereditarily separable.

### Problems I wish I could solve by S. Watson

**Problem 69.** A. Dow [92] showed that it is consistent and independent of CH that every normal space of character at most  $\mathfrak{c}^+$  is collectionwise Hausdorff. This provides a negative answer to both Problems 69 and 70.

**Problem 70.** No, see Problem 69.

**Problem 77.** See F.D. Tall's Problem 48.

**Problem 84.** No, D. Shakhmatov, F.D. Tall and S. Watson [248] constructed a consistent example of a normal Moore space which is not submetrizable. Whether a positive answer can be established without using a large cardinal remains open. Also, Tall [271] has shown that under the assumption of a supercompact cardinal, there is a model of set theory in which all normal Moore spaces are submetrizable, but in which there exist nonmetrizable normal Moore spaces.

**Problem 85.** See F.D. Tall's Problem 43.

**Problem 86.** *Are countably paracompact, locally compact, metacompact spaces paracompact?* The counterexample of G. Gruenhage and P. Koszmider to the Arhangel'skii–Tall problem (see Problem 43) is countably paracompact.

**Problem 87.** Yes. G. Gruenhage and P. Koszmider [127] showed that, under  $\text{MA}_{\aleph_1}$ , normal, locally compact, meta-Lindelöf spaces are paracompact.

**Problem 88.** *Does ZFC imply that there is a perfectly normal, locally compact space which is not paracompact?* P. Larson and F.D. Tall [173] proved that if it is consistent that there is a supercompact cardinal, then it is consistent that every perfectly normal, locally compact space is paracompact.

**Problem 92.** *Are normal, locally compact, locally connected spaces collectionwise normal?* Z. Balogh [18] showed that it is consistent, relative to the existence of a compact cardinal, that locally compact, normal spaces are collectionwise normal. It remains open whether large cardinals are needed to establish a positive answer.

**Problem 94.** *Does  $2^{\aleph_0} < 2^{\aleph_1}$  imply that separable, first countable, countably paracompact spaces are collectionwise Hausdorff?* R. Knight [159] announced that there is model of  $2^{\aleph_0} < 2^{\aleph_1}$  such that there exists an uncountable subset of  $\mathbb{R}$  that is a  $\Delta$ -set. Note that in such a model there must be dominating family in  ${}^{\omega_1}\omega$  of cardinality  $2^\omega$ . The cardinal arithmetic of Knight's model seems to be flexible enough to obtain a one-to-one continuum function. (The topological example is the tangent-disk space over this subset of  $\mathbb{R}$ ; such a tangent-disk space is countably paracompact iff the subset is a  $\Delta$ -set.) This provides negative answers to both Problems 94 and 96. In [158], Knight constructed a model of ZFC in which there is a subset of  $\mathbb{R}$  that is  $\Delta$ -set but not a  $Q$ -set.

**Problem 96.** *If the continuum function is one-to-one and  $X$  is a countably paracompact, first countable space, then is  $e(X) \leq c(X)$ ? No, see Problem 94.*

**Problem 97.** *Does  $\diamond^*$  imply that countably paracompact, first countable spaces are  $\aleph_1$ -collectionwise Hausdorff? Yes, K. Smith and P. Szeptycki [264] showed that, assuming  $\diamond^*$ , paranormal spaces of character  $\leq \aleph_1$  are  $\omega_1$ -collectionwise Hausdorff. A space is *paranormal* if every countable discrete collection of closed sets can be expanded to a locally finite collection of open sets. Both countably paracompact spaces and normal spaces are paranormal.*

**Problem 99.** No, P. Nyikos [212] claimed that if there is a Souslin tree then there is a collectionwise Hausdorff, Aronszajn tree which is not countably paracompact.

**Problem 104.** W. Fleissner [119, §3] gave a repaired construction of the space Son of George, which is what Watson really wanted in Problem 104.

**Problem 110.** *Is it consistent that meta-Lindelöf, collectionwise normal spaces are paracompact? No. Z. Balogh [21] constructed a hereditarily meta-Lindelöf, hereditarily collectionwise normal, hereditarily realcompact Dowker space. Balogh also constructed a meta-Lindelöf, collectionwise normal, countably paracompact space which is not metacompact.*

**Problem 113.** Z. Balogh gave a ZFC construction of a Dowker space which is hereditarily normal and scattered of height  $\omega$ . This gives affirmative answers to Problems 113, 114, and 115, which ask for a ZFC example of a Dowker space that is, respectively, hereditarily normal,  $\sigma$ -discrete, and scattered. See [23] for an exposition of Balogh's technique.

**Problem 114.** See Problem 113.

**Problem 115.** See Problem 113.

**Problem 116.** Yes (Z. Balogh [20]). See Problem 57.

**Problem 132.** Under CH, W.L. Saltzman [243] constructed a nondegenerate connected CDH subset of the plane which has a rigid open subset.

**Problem 134.** Under CH, W.L. Saltzman [242] constructed a connected CDH subset of the plane which is not SLH.

**Problem 141.** In the discussion before the statement of Problem 142, Watson stated that if one forces with a Souslin tree, then one can make a collectionwise normal space into a nonnormal space. Watson now retracts such a claim and this becomes Problem 141 $\frac{1}{2}$ .

**Problem 142.** *Can Cohen forcing make a collectionwise normal space not normal? W. Fleissner, T. LaBerge and A. Stanley [120] described a construction that takes any*

normal space  $X$  and outputs a normal superspace  $T$  such that  $T$  becomes nonnormal after adding one Cohen real if and only if  $X$  is a Dowker space. A similar construction applied to Rudin's box product Dowker space yields a collectionwise normal space that becomes nonnormal after the addition of one Cohen real. This provides negative answers to Problems 142, 143 and 144. R. Grunberg, L. Junqueira and F.D. Tall [129] showed that if  $X$  is normal but not normal after adding one Cohen real then  $X$  is a Dowker space.

**Problem 143.** *Can one Cohen real kill normality?* See Problem 142.

**Problem 144.** *Is there, in ZFC, a c.c.c. partial order which kills collectionwise normality?* Yes. R. Grunberg, L. Junqueira and F.D. Tall [129] showed that any strengthening of the topology on the real line which is locally compact, locally countable, separable and collectionwise normal is an example of a collectionwise normal space which can be made nonnormal by c.c.c. forcing. The Eric (van Douwen) Line is such a strengthening.

**Problem 145.** *Can countably closed, cardinal-preserving forcing make a nonnormal space normal?* R. Grunberg, L. Junqueira and F.D. Tall [129] gave a consistent answer. Suppose there is an uncountable regular  $\kappa$  such that  $\kappa^{<\kappa} = \kappa$ . Then there is a nonnormal space  $X$  and a countably closed, cardinal-preserving  $\mathcal{P}$  such that  $\mathcal{P}$  forces  $X$  to be normal.

**Problem 146.** *Can c.c.c. forcing make a nonnormal space metrizable?* Yes, W. Fleissner produced, in ZFC, a c.c.c. forcing which turns a nonnormal space into a metrizable space.

**Problem 147.** *Is there, in ZFC, a cardinal-preserving forcing which makes a nonnormal space metrizable?* See Problem 146.

**Problem 149.** *Does countably closed forcing preserve hereditary normality?* No. R. Grunberg, L. Junqueira and F.D. Tall [129] showed that adding a Cohen subset of  $\omega_1$  with countable conditions will destroy the normality of a non- $\aleph_1$ -collectionwise Hausdorff space. In particular, this countably closed forcing does not preserve the hereditary normality of Bing's Example G.

**Problem 151.** A. Dow [91, Theorem 7.28] proved that it is possible to lower the density of a space with cardinal-preserving forcing. Dow's method uses a measurable cardinal.

**Problem 153.** A. Dow [91, Theorem 7.29] proved that it is possible to make a regular (or a first countable Hausdorff) non-Lindelöf space Lindelöf with cardinal-preserving forcing. Dow's method uses a measurable cardinal.

**Problem 160.** D. Shakhmatov and M. Tkachenko [249] proved that the existence of a compact Hausdorff space of size  $2^{\aleph_0}$  that is  $T_1$ -complementary to itself is both consistent with and independent of ZFC. They also constructed, in ZFC, a countably compact Tychonoff space of size  $2^{\aleph_0}$  which is  $T_1$ -complementary to itself and a compact Hausdorff space of size  $2^{\aleph_0}$  which is  $T_1$ -complementary to a countably compact Tychonoff space. This provides complete solutions to Problems 160 and 161.

**Problem 161.** See Problem 160. The existence of two infinite  $T_1$ -complementary compact Hausdorff spaces was announced to Watson by B. Aniszczyk in 1989, but the example has never been published.

**Problem 162.** M. Tkačenko, V. Tkachuk, R. Wilson and I. Yashchenko [273] proved that proved that no  $T_1$ -complementary topology exists for the maximal topology constructed by E.K. van Douwen on the rational numbers.

**Problem 167.** *Which topology on a set of size  $n$  has the largest number of complements?* The natural conjecture is that the partial order ( $T_0$  topology) with the least number of complements is the partial order made up of an antichain and two comparable elements, the partial order ( $T_0$  topology) with the greatest number of complements is the partial order made up an antichain and a maximum and a minimum. These conjectures remain open although J.I. Brown and S. Watson have shown they are asymptotically correct. See [44–46].

**Problem 172.** Yes, J. Harding and A. Pogel [135] proved that every lattice with 1 and 0 is embeddable in the lattice of topologies of some set.

**Problem 175.** The problem should have stated “open intervals” instead of “open sets”. Y.-Q. Qiao and F.D. Tall [272] showed that the existence of a linear ordering as correctly stated is equivalent to the existence of a perfectly normal nonmetrizable non-Archimedean space (i.e., an archvillain). See Problem 374. Qiao [225] showed that there is a model of  $\text{MA} + \neg\text{CH}$  in which there is an archvillain (and yet no Souslin lines); this answers the second half of the problem negatively.

**Problem 176.** *Is there a topological space in which the nondegenerate connected sets are precisely the cofinite sets?* G. Gruenhage [125] gave several consistent examples. Assuming  $\text{MA}$ , there are completely regular as well as countable examples. Assuming  $\text{CH}$ , there is a perfectly normal example.

### Weiss’s questions by W. Weiss

**Problem 179.** P. Koszmider [161] proved that there is an uncountable product of nontrivial compact convex subsets of normed linear spaces that fails to have the complete invariance property.

**Problem 180.** *Is there a bound on the size of countably compact, locally countable, regular spaces?* Such spaces are called *good*; if, in addition, separable subspaces are at most countable then they are called *splendid*. I. Juhász, Z. Nagy and W. Weiss [141] proved that there are splendid spaces of size  $\aleph_n$  for each  $n \in \omega$ , and if  $\mathbf{V} = \mathbf{L}$  then there are arbitrarily large splendid spaces. P. Nyikos observed that the  $\mathbf{V} = \mathbf{L}$  results only require that the covering lemma holds over the Core Model and various other weak hypotheses. On the other hand, I. Juhász, S. Shelah and L. Soukup [142] showed that if the Chang Conjecture

variant  $(\aleph_{\omega+1}, \aleph_\omega) \rightarrow (\omega_1, \omega)$  holds then there are no splendid spaces of cardinality greater than  $\aleph_\omega$ . Also, ZFC is enough to show that there are none of cardinality exactly equal to  $\aleph_\omega$  or any other singular cardinal of countable cofinality. In fact this is the hurdle for which something beyond ZFC is needed: in ZFC one can easily go from  $\aleph_n$  to  $\aleph_{n+1}$  but the jump at  $\aleph_\omega$  causes major complications. The Chang Conjecture variant has nothing to say about good spaces and the problem for good spaces is still open. It would entail solving another problem: *Is there a regular separable, locally countable, countably compact, noncompact space?* This is a special case of the title problem of the section by P. Nyikos and now carries a reward of a US\$1000: *Is there a ZFC example of a separable, first countable, countably compact, noncompact Hausdorff space?*

**Problem 181.** *Let  $X$  be a regular space and let  $\lambda$  be the least cardinal such that some open cover of size  $\lambda$  has no finite subcover and let  $\kappa$  be the least cardinal such that every point has a neighbourhood of size  $< \kappa$ . How are  $\lambda$ ,  $\kappa$  and  $|X|$  related?* P. Nyikos remarks that if  $\kappa = \lambda$  is strongly inaccessible,  $|X|$  can be arbitrarily large. Take  $D$  discrete as big as you wish and let  $S$  be the space of ultrafilters  $\mu \in \beta D$  for which there is  $E \in \mu$  with  $|E| < \lambda$ . A similar example works whenever  $\kappa > 2^{2^{<\lambda}}$ .

**Problem 185** (A. Hajnal and I. Juhász). *Does each Lindelöf space of cardinality  $\aleph_2$  have a Lindelöf subspace of cardinality  $\aleph_1$ ?* P. Koszmider and F.D. Tall [164] used countably closed forcing to construct an example of a subspace of the countable box topology on the product of  $\aleph_2$  copies of the two-point discrete space. Their example is an uncountable Lindelöf  $T_2$  P-space with no Lindelöf subspaces of cardinality  $\aleph_1$  (actually, with no convergent  $\omega_1$ -sequences). They showed that their construction will not work ZFC.

### Perfectly normal compacta, cosmic spaces, and some partition problems by G. Gruenhage

There have been no solutions to the six problems in this section.

### Open problems on $\beta\omega$ by K.P. Hart, J. van Mill

**Problem 201.** Yes. S. Shelah and J. Steprāns [259] showed that it is consistent with  $\text{MA} + \neg\text{CH}$  that a totally nontrivial (= nowhere trivial) automorphism exists.

**Problem 205.** The answer is independent. This is the same as Problem 30.

**Problem 210.** The answer is independent. On the one hand it is consistent with  $\text{MA} + \neg\text{CH}$  that every autohomeomorphism of  $\omega^*$  is trivial [257]. In this model the answer is negative: there are  $\mathfrak{c}$  autohomeomorphisms but  $2^{\mathfrak{c}}$   $P_{\mathfrak{c}}$ -points. On the other hand, J. Steprāns [268] showed the consistency of a positive answer.



**Problem 216.** No. A. Dow [95] showed that it is consistent with  $\neg\text{CH}$  that closed subsets of the space  $\omega^*$  are exactly the compact zero-dimensional  $F$ -spaces of weight  $\leq c$ .

**Problem 217.** This is a special case of Problem 9, which was solved in the negative by A. Dow and J. Vermeer. See Problem 9.

**Problem 221.** *Is every nowhere dense set in  $\omega^*$  a  $c$ -set?* See Problem 222.

**Problem 222.** *Is there a maximal nowhere dense subset in  $\omega^*$ ?* In the book, it was noted that no  $c$ -set can be a maximal nowhere dense set and that it is consistent that every nowhere dense set is a  $c$ -set (see [16]). P. Simon [262] showed that Problem 221 and Problem 222 are equivalent: every nowhere dense set in  $\omega^*$  is a  $c$ -set if and only if there are no maximal nowhere dense subsets in  $\omega^*$ . In 1975, A.I. Veksler [286] had shown that nowhere dense  $P$ -sets are not maximal.

**Problem 223.** A. Bella, A. Błaszczyk and A. Szymański [33] proved that if  $X$  is compact, extremally disconnected, without isolated points and of  $\pi$ -weight  $\aleph_1$  or less then  $X$  is an AR for extremally disconnected spaces iff  $X$  is the absolute of one of the following three spaces: the Cantor set, the Cantor cube  ${}^{\omega_1}2$ , or the sum of these two spaces. This provides a negative answer to Problem 223 under CH.

**Problem 226.** *Is it consistent that there is, up to permutation, only one  $P$ -point in  $\omega^*$ ?* Yes. See S. Shelah [251, XVIII §4].

**Problem 228.** *Is there is a  $p$  in  $\omega^*$  such that every compactification of  $\omega \cup \{p\}$  contains  $\omega^*$ ?* Yes (A. Dow [101]). Take a map  $f$  from  $\beta\omega$  onto  $I^c$ , take a closed set  $A$  such that  $f \upharpoonright A$  is irreducible and finally take any  $p$  in  $A$ .

**Problem 229.** This problem was partially solved by E. Coplakova and K.P. Hart [69]. They proved that if the bounding number  $b$  equals  $c$  then there exists a point  $p$  in  $\mathbb{Q}^*$  (the Čech–Stone remainder of the space of rational numbers) such that  $p$  generates an ultrafilter in the set-theoretic sense on  $\mathbb{Q}$  and such that  $p$  has a base consisting of sets that are homeomorphic to  $\mathbb{Q}$ .

**Problem 231.** ( $\text{MA} + \neg\text{CH}$ ) *Are there a Hausdorff gap  $\mathbb{G} = \langle (f_\alpha, g_\alpha) : \alpha \in \omega_1 \rangle$  and a ( $P$ -point, selective ultrafilter)  $p$  such that  $p \subseteq I_{\mathbb{G}}^+$ ?* Here  $I_{\mathbb{G}}$  is the induced gap-ideal, i.e., the ideal on  $\omega$  of subsets over which the gap is filled,  $I_{\mathbb{G}} = \{M : (\exists h \in {}^M\omega)(\forall \alpha) f_\alpha \upharpoonright M <^* h <^* g_\alpha \upharpoonright M\}$ . S. Kamo [145] proved that if  $V$  is obtained from a model of CH by adding Cohen reals then in  $V$  an ideal is a gap-ideal iff it is  $\leq \omega_1$ -generated. Also, CH implies that any nontrivial ideal is a gap-ideal. In a preprint, Kamo [146] showed, under  $\text{MA} + \neg\text{CH}$ , that for every Hausdorff gap  $\mathbb{G}$  there are both selective ultrafilters and non- $P$ -points consisting of positive sets (with respect to the gap-ideal  $I_{\mathbb{G}}$ ). Also, under  $\text{MA} + \neg\text{CH}$ , there is a selective non- $P_{\omega_2}$ -point that meets every gap-ideal.

**Problem 237.** D. Strauss [269] showed that  $\langle \beta\mathbb{N}, + \rangle$  cannot be embedded in  $\langle \mathbb{N}^*, + \rangle$ . Specifically, if  $\phi: \beta\mathbb{N} \rightarrow \mathbb{N}^*$  is a continuous homomorphism then the image of  $\phi$  must be finite.

**Problem 240.** Yes. I. Farah [117] proved a generalization of Problems 240 and 241: Assume  $Z$  is a  $\beta\mathbb{N}$ -space,  $X$  is compact,  $\kappa$  is an arbitrary cardinal and  $f: X^\kappa \rightarrow Z$ . Then  $X^\kappa$  can be covered by finitely many clopen rectangles such that  $f$  depends on at most one coordinate on each one of them.

**Problem 241.** See Problem 240.

**Problem 244.** S. Shelah and O. Spinas [255] proved that for every  $n$  one can have a model in which  $wn((\omega^*)^n) > wn((\omega^*)^{n+1})$ . This provides some information about Problem 244.

**Problem 245.** Yes, to the second part of the problem. S. Shelah and O. Spinas [256] showed that  $wn(\omega^*) > wn(\omega^* \times \omega^*)$  is consistent.

**Problem 264.** This problem is solved. A result due to A. Dow [89] shows that under  $\neg\text{CH}$  there are always  $p$  and  $q$  for which  $\mathbb{I}_p$  and  $\mathbb{I}_q$  are not homeomorphic. A. Dow and K.P. Hart [96] showed that under CH any two continua  $\mathbb{I}_p$  and  $\mathbb{I}_q$  are homeomorphic. It follows that the statement *all continua  $\mathbb{I}_p$  are homeomorphic* is equivalent to CH.

**Problem 265.** *Are there cutpoints in  $\mathbb{I}_p$  other than the points  $f_p$  for  $f: \omega \rightarrow \mathbb{I}$ ?* This problem is solved; as indicated in the paper the answer is yes under  $\text{MA}_{\text{countable}}$  [17]. A. Dow and K.P. Hart [97] confirmed the conjecture that the answer is no in Laver's model for the Borel Conjecture.

**Problem 266.** A. Dow and K.P. Hart [98] have shown that there are least 14 different subcontinua of  $\beta\mathbb{R} \setminus \mathbb{R}$ : 10 in ZFC alone, four more under CH or at least six more under  $\neg\text{CH}$ .

### On first countable, countably compact spaces III by P.J. Nyikos

**Problem 286.** No. T. Eisworth and J. Roitman [116] showed that CH is not enough to imply the existence of an Ostaszewski space.

**Problem 287.** Yes. T. Eisworth [114] showed that it is consistent with CH that first countable, countably compact spaces with no uncountable free sequences are compact. Consequently, it is consistent with CH that perfectly normal, countably compact spaces are compact.

**Problem 292.** M. Rabus [226] proved that it is consistent with MA and  $\mathfrak{t} = \mathfrak{K}_2 = \mathfrak{c}$  that every  $\subset^*$ -increasing  $\omega_1$ -sequence in  $\mathcal{P}(\omega)$  is the bottom part of some tight  $(\omega_1, \omega_2^*)$ -gap. In the discussion after Axiom 5.6 (p. 151), P. Nyikos wrote: "Of course, the really

interesting models are those where  $\mathfrak{b} < \mathfrak{c}$ , and there Problem 10 (= Problem 292) and its analogue for higher  $\kappa$  ( $> \omega_1$ ) seem to be completely open". Z. Spasojević [267] answered these questions by providing such models. Spasojević thereby provided new models (where  $\mathfrak{b} < \mathfrak{c}$ ) which contain separable, first countable, countably compact, noncompact Hausdorff spaces. The existence of such spaces is the (still open in ZFC) title problem of Nyikos's article.

**Problem 296.** Z. Spasojević [267] showed that  $\mathfrak{p} = \omega_1$  implies that there is a tight  $(\omega_1, \omega_1^*)$ -gap in  ${}^{\mathbb{N}}\mathbb{N}$ , according to Definition 6.8 (p. 157) by P. Nyikos. However, Nyikos misstated the definition of a tight gap for a pair of families  $A, B$  in  ${}^{\mathbb{N}}\mathbb{N}$ . Definition 6.8 should have specified that pair  $A, B$  has to be a gap in  ${}^{\mathbb{N}}\mathbb{N}$  as well. That is,  $f <^* g$  for each  $f$  in  $A, g$  in  $B$ . Problem 296, with this corrected definition, is still open. It is this corrected version that is needed for the construction of a separable, countably compact, noncompact manifold.

### Set-theoretic problems in Moore spaces by G.M. Reed

**Problem 298.** This is the same as Problem 84.

**Problem 299** (F.B. Jones). *Is it consistent with ZFC that the square of each normal Moore space is normal?* H. Cook had given an example under  $\text{MA} + \neg\text{CH}$ ; this example, and others by Reed were published in [68]. Beyond  $\text{MA} + \neg\text{CH}$ , it was known that the existence of normal, non-collectionwise Hausdorff, Moore sequence spaces is consistent with GCH. Reed showed that existence of a normal, non-collectionwise Hausdorff, Moore sequence space implies the existence of a normal Moore space whose square is not normal.

**Problem 300.** Reed showed that it is consistent with ZFC that there exists a normal, locally compact, separable Moore space  $X$  such that  $X^2$  is not normal. Reed's result was announced in [68].

**Problem 303.** No, surprisingly. D. Fearnley [118] constructed a Moore space with a  $\sigma$ -discrete  $\pi$ -base which cannot be densely embedded in any Moore space with the Baire property.

**Problem 305.** *Is each starcompact Moore space compact?* Yes if CH (G.M. Reed and A.W. Roscoe). Specifically, if  $X$  is a starcompact Moore space that is not compact then  $\mathfrak{d} < w(X) < \mathfrak{c}$ . See [88].

**Problem 306.** Yes. S. Todorćević [275] constructed a first countable, zero-dimensional space  $X$  such that  $X^2$  has an uncountable point-finite family of open sets but  $X$  itself does not have such a family, i.e.,  $X$  has caliber  $(\omega_1, \omega)$  but  $X^2$  does not have caliber  $(\omega_1, \omega)$ .

**Problem 310.** *Can each locally compact, separable Moore space be densely embedded in a pseudocompact Moore space?* P. Nyikos announced a positive solution in 1991. Independently, P. Simon and G. Tironi [263] proved a positive solution too.

**Problem 314.** It is consistent that the answer is negative. I. Tree and S. Watson [280] gave an example, under CH, of a nonmetrizable pseudocompact Moore manifold. It is not known whether Problem 314 has a negative answer in ZFC. Also, Tree gave a ZFC example of a nonmetrizable pseudonormal Moore manifold (which is listed as a subproblem to Problem 314). Reed would like to note that P. Nyikos had produced an example of a pseudonormal, nonmetrizable Moore manifold several years ago, which Reed had forgotten.

**Problem 315.** Reed has shown that each star-refining-paracompact Moore space is countably paracompact, and he has an example of a Moore space in which each open cover has a  $\sigma$ -locally finite star-refinement, but which is not strongly star-refining-screenable. It remains an open question as to whether it is consistent with ZFC that each star-refining-paracompact Moore space is metrizable.

#### Some conjectures by M.E. Rudin

**Problem 318.** Yes is consistent. Z. Balogh and G. Gruenhage [24] proved the consistency of the existence of a Dowker filter on  $\omega_2$ .

**Problem 319.** Z. Balogh [19] gave a ZFC example of a Dowker space of cardinality  $\mathfrak{c}$ , answering Rudin's alternate problem. Using pcf theory in ZFC, M. Kojman and S. Shelah [160] showed that there is Dowker space of cardinality  $\aleph_{\omega+1}$ .

**Problem 320.** Z. Balogh [20] constructed a normal screenable nonparacompact space in ZFC. This is a partial answer to Problem 320 (screenable rather than  $\sigma$ -disjoint base). See Problem 57.

**Problem 324.** Yes. Z. Balogh [22] proved that for every uncountable cardinal  $\kappa$  there is a space  $X$  such that the product of  $X$  with every metrizable space is normal and  $X$  has an increasing  $\omega_1$ -cover with no refinement by fewer than  $\kappa$  closed subsets of  $X$ . This implies a positive answer to Problem 324 and proves the second (and thus all three) of K. Morita's duality conjectures.

**Problem 329** (Michael's Conjecture). *There is a Michael space.* That is, there is a Lindelöf space whose product with the irrationals is not Lindelöf (or, equivalently, not normal). J. Moore [203] proved that it is consistent that there is a Michael space of weight less than  $\mathfrak{b}$ . Moore also proved that  $\mathfrak{d} = \text{cov}(\text{Meager})$  implies that there is a Michael space.

**Problem 331.** No. B. Lawrence [174] proved that the box product of  $\omega_1$  copies of  $(\omega + 1)$  is neither normal nor collectionwise Hausdorff.

### Small uncountable cardinals and topology by J.E. Vaughan

**Problem 333.** S. Shelah has solved some of the oldest problems on cardinal invariants of the continuum. In [250], Shelah proved the consistency of  $i < u$ . In [253], Shelah proved that  $\mathfrak{a} > \mathfrak{d}$  is consistent and also that  $\mathfrak{a} > u$  is consistent (using a measurable cardinal). See [42] for an exposition of the technique of iterations along templates. Recently, Shelah [254] announced that  $\mathfrak{p} < \mathfrak{t}$  is consistent. The only remaining open basic question about small cardinals is whether  $i < \mathfrak{a}$  is consistent. Also, it is not known whether  $\mathfrak{a} > \mathfrak{d} = \mathfrak{N}_1$  is consistent.

**Problem 334.** *Can  $\mathfrak{a}$  or  $\mathfrak{s}$  be singular?* S. Shelah [253] proved that  $\mathfrak{a}$  can be singular of uncountable cofinality. J. Brendle [43] proved that  $\mathfrak{a}$  can be any singular cardinal of countable cofinality.

**Problem 337.** *Can  $\text{cf}(\text{cov}(\mathbb{L})) = \omega$ ?* Yes, S. Shelah [252] proved that it is consistent that  $\text{cf}(\text{cov}(\mathbb{L})) = \aleph_\omega$ . A. Miller had proved that  $\text{cov}(\mathbb{K})$  cannot have countable cofinality and this was improved by T. Bartoszynski and H. Judah to  $\text{cf}(\text{cov}(\mathbb{K})) \geq \text{add}(\mathbb{L})$ . ( $\mathbb{L}$  is the ideal of null sets,  $\mathbb{K}$  is the ideal of meager sets.)

**Problem 339.** *Is  $\mathfrak{t} \leq \text{add}(\mathbb{L})$ ?* It was known that there are models of  $\mathfrak{p} = \mathfrak{c} > \omega_1 = \text{cov}(\mathbb{L})$  (in which case  $\mathfrak{p} = \mathfrak{t} = \mathfrak{c}$  and  $\text{cov}(\mathbb{L}) = \text{add}(\mathbb{L}) = \omega_1$ ). Several methods for constructing such models were mentioned in the second status report [194].

**Problem 343.** No is consistent. A. Dow [90, §5] showed that there is a model in which there is a noncountably-compact product of  $\mathfrak{h}$  sequentially compact spaces.

**Problem 345.** P. Nyikos has withdrawn the claim that there is a compact Hausdorff space of cardinality  $2^{\mathfrak{s}}$  with no nontrivial convergent sequences.

**Problem 354.** P. Koszmider [162] has settled this problem. He proved that it is consistent that there is a normal, first countable, noncompact, initially  $\omega_1$ -compact space.

**Problem 359.** *What is  $\mathfrak{a}_p = \{|X| : X \text{ is first countable and pseudocompact but not countably compact}\}$ ?* I. Tree observed that it is consistent that  $\mathfrak{a}_p < \mathfrak{a}$ .

Vaughan's article discusses the ten questions about small cardinals from van Douwen's article [86]. A report on these questions also appears in van Douwen's collected papers [87].

### A survey of the class MOBI by H.R. Bennett, J. Chaber

There have been no solutions to the eleven problems listed in this section.

### Problems in perfect ordered spaces by H.R. Bennett, D.J. Lutzer

**Problem 373.** *Is it true that a perfect generalized ordered space can be embedded in a perfect linearly ordered space?* W.-X. Shi [260] proved that any perfect generalized ordered space with a  $\sigma$ -closed-discrete dense set can be embedded in a perfect linearly ordered space.

**Problem 374.** This problem has several equivalent versions.

- (Nyikos) Is there is a perfectly normal, non-Archimedean, nonmetrizable space?
- (Problem 374, Maurice and van Wouwe) Is there is a perfect linearly ordered space which does not have a  $\sigma$ -discrete dense subspace?
- (Problem 175) Is there is a linearly ordered space in which every disjoint collection of convex open sets is  $\sigma$ -discrete, but which does not have a  $\sigma$ -discrete dense subspace?
- (Tall and Qiao) Is there is a linearly ordered space without isolated points which does not have a  $\sigma$ -discrete dense subspace, but every nowhere dense subspace of it does have such a subspace?

**Problem 376.** No. W.-X. Shi [261] constructed a nonmetrizable, compact, linearly ordered topological space such that every subspace has a  $\sigma$ -minimal base for its relative topology. Bennett and Lutzer [35] had constructed an example that was Čech complete, perfect and paracompact but not compact.

### The point-countable base problem by P.J. Collins, G.M. Reed, A.W. Roscoe

**Problem 381.** No. M.E. Rudin [234] constructed a monotonically normal space that is not  $K_0$ , hence not acyclically monotonically normal.

### Some open problems in densely homogeneous spaces by B. Fitzpatrick Jr, H.-X. Zhou

**Problem 382.** See Problem 134.

**Problem 384.** Under CH, W.L. Saltzman [243] constructed a connected CDH subset of the plane which has a rigid open subset.

**Problem 387.** *For which zero-dimensional subsets  $X$  of  $\mathbb{R}$  is  $X^\omega$  homogeneous? CDH?* B. Lawrence [175] proved that all zero-dimensional subsets of  $\mathbb{R}$  have a homogeneous  $\omega$ -power. A. Dow and E. Pearl [99] proved that all zero-dimensional, first countable spaces have a homogeneous  $\omega$ -power. M. Hrusak and B. Zamora-Avilés showed that if  $X$  is a zero-dimensional Borel subset of  $\mathbb{R}$  then  $X^\omega$  is CDH if and only if  $X$  is a  $G_\delta$  set.

**Problem 389.** *Does there exist a CDH metric space that is not completely metrizable? Yes if MA or if CH is known. M. Hrusak and B. Zamora-Avilés proved that if  $X$  is metrizable, Borel and a CDH space, then  $X$  is completely metrizable.*

**Problem 390.** *Is there an absolute example of a CDH metric space of cardinality  $\aleph_1$ ? This problem is still open. Specifically, is there a CDH subset of  $\mathbb{R}$  of size  $\aleph_1$ ? Yes if MA or if CH.*

### Large homogeneous compact spaces by K. Kunen

**Problem 391.** van Douwen's Problem is still open: *Is there is a compact homogeneous space with cellularity greater than  $\mathfrak{c}$ ?*

### Some problems by E. Michael

**Problem 392.** *Let  $f : X \rightarrow Y$  be a continuous map from a separable metrizable space  $X$  onto a metrizable space  $Y$ , with each fiber  $f^{-1}(y)$  compact.*

- (a) *If  $f$  is countable-compact-covering, must  $f$  be compact-covering?*
- (b) *If  $f$  is compact-covering, must  $f$  be inductively perfect?*

G. Debs and J. Saint Raymond gave negative answers to both parts of this question. For (b), they gave a negative answer in [76, Theorem 7.13]. For (a), they gave a negative answer in [77, Theorem 7.2] with a map  $f$  whose fibers  $f^{-1}(y)$  are actually finite (and whose domain is  $\sigma$ -compact). However, they showed in [77, Corollary 6.5] that the answer to both (a) and (b) becomes positive if it is assumed that, for some  $n$ , all fibers of  $f$  have at most  $n$  elements.

**Problem 393.** *Let  $f : X \rightarrow Y$  be a continuous map from a separable metrizable space  $X$  onto a countable metrizable space  $Y$ . If  $f$  is compact-covering, must  $f$  be inductively perfect? A.V. Ostrovskii [216] gave a positive answer, even when  $Y$  is only assumed to be  $\sigma$ -compact and metrizable.*

**Problem 394.** *If a Hausdorff space  $Y$  is a quotient  $s$ -image of a metric space, must  $Y$  be a compact-covering quotient  $s$ -image of a (possibly different) metric space? No, H. Chen [63] constructed a Hausdorff counterexample. E. Michael then asked whether there exists a such an example which is regular or even paracompact, and Chen [64] then showed that it is consistent that there is a regular counterexample.*

**Problem 395.** Remark 3.9 on p. 276 the last sentence should say: If the answer to Question 3.2 (= Problem 395) is “yes”, and if  $B$  is separable, then such a  $f$  exists even when  $X$  carries the weak\* topology and  $Y$  the norm topology.

**Problem 396.** *If  $X$  be paracompact,  $Y$  a Banach space, and  $K$  a convex  $G_\delta$ -subset of  $Y$ . Must every l.s.c. mapping from  $X$  to the space of nonempty, convex, relatively closed subsets of  $K$  have a continuous selection?* V. Gutev and V. Valov [133] gave a positive answer in case  $X$  is a  $C$ -space. Previously, Gutev [132] had obtained a positive answer under the stronger assumption that  $X$  is either a countable-dimensional metric space or a strongly countable-dimensional paracompact space.

### Questions in dimension theory by R. Pol

**Problem 398.** Consistently, the gap between the inductive dimensions for nonseparable metrizable spaces can be arbitrarily large. See Problem 399.

**Problem 399.** S. Mrówka [207] constructed an example of a zero-dimensional metrizable space, called  $\nu\mu_0$ , such that under a particular set-theoretic axiom  $S(\aleph_0)$ ,  $\nu\mu_0$  does not have a zero-dimensional completion. Specifically, under  $S(\aleph_0)$  each completion of  $\nu\mu_0$  contains a copy of the interval  $[0, 1]$ . In particular,  $\text{ind } \nu\mu_0 = 0$  and, under  $S(\aleph_0)$ ,  $\dim \nu\mu_0 = 1$ . Mrówka [208] extended this result to show that under  $S(\aleph_0)$ , any completion of  $(\nu\mu_0)^2$  contains copy of the square  $[0, 1]^2$ . J. Kulesza [169] generalized this by showing that under  $S(\aleph_0)$ , every completion of  $(\nu\mu_0)^n$  contains an  $n$ -cube. In particular,  $\text{Ind}(\nu\mu_0)^n = \dim(\nu\mu_0)^n = n$  under  $S(\aleph_0)$ . This provides answers to Problems 398 and 399. R. Dougherty [85] proved the relative consistency of the set-theoretic axiom  $S(\aleph_0)$ .  $S(\aleph_0)$  has roughly the strength of an Erdős cardinal. Specifically, Dougherty proved that from the Erdős cardinal  $E(\omega_1 + \omega)$ ,  $S(\aleph_0)$  is consistent and that from  $S(\aleph_0)$ , it is consistent that  $E(\omega)$  exists.

**Problem 407.** This problem was solved by J. Dydak and J.J. Walsh. See Problem 649.

**Problem 418.** *What is the compactness degree  $\text{cmp}$  of the space  $J_n = [0, 1]^{n+1} \setminus \{0\} \times (0, 1)^n$ ?* J. de Groot and T. Nishiura asked if  $\text{cmp } J_n \geq n$  for  $n \geq 3$ . V.A. Chatyrko and Y. Hattori [62] proved that if  $n \leq 2^m - 1$  for some integer  $m$ , then  $\text{cmp } J_n \leq m + 1$ . In particular,  $\text{cmp } J_n < \text{def } J_n$  for  $n \leq 5$ . ( $\text{def}$  is the *compactness deficiency*.) Subsequently, Nishiura proved  $\text{cmp } J_4 < \text{def } J_4$ . The original problem is still open for  $n = 3$ . Furthermore, Aarts and Nishiura had asked for examples to witness any possible values of  $\text{cmp} \leq \text{def}$ . Recently, Chatyrko proved that for any positive integers  $k$  and  $m$  such that  $k \leq m$  there exists a separable metrizable space  $X(k, m)$  such that  $\text{cmp } X(k, m) = k$  and  $\text{def } X(k, m) = m$ .

**Problem 423.** This problem, as it appears in the book, was solved by A.N. Dranishnikov and V.V. Uspenskij [104]. Pol informed Uspenskij that the problem should have been posed differently.

**Problem 423.** *Let  $f : X \rightarrow Y$  be a continuous map of a compactum  $X$  onto a compactum  $Y$  with  $\dim f^{-1}(y) = 0$  for all  $y \in Y$ . Let  $A$  be the set of all maps  $u : X \rightarrow \mathbb{I}$  into the unit interval such that  $u[f^{-1}(y)]$  is zero-dimensional for all  $y \in Y$ . Do almost all maps*



$u : X \rightarrow \mathbb{I}$ , in the sense of Baire category, belong to  $A$ ? ( $X, Y$  are compact separable metrizable spaces.)

H. Toruńczyk had given a positive answer under the assumption that  $Y$  is countable-dimensional. Uspenskij [285] extended this result to the case when  $Y$  is a  $C$ -space. M. Tuncali and V. Valov [282] further extended this result: Let  $f : X \rightarrow Y$  be a  $\sigma$ -perfect surjection such that  $\dim f \leq n$  and  $Y$  is a paracompact  $C$ -space. Let  $\mathcal{H} = \{g \in C(X, \mathbb{I}^{n+1}) : \dim g(f^{-1}(y)) \leq n \text{ for each } y \in Y\}$ . Then  $\mathcal{H}$  is dense and  $G_\delta$  in  $C(X, \mathbb{I}^{n+1})$  with respect to the source limitation topology. In the general case, the revised problem remains open.

**Problem 424.** Let  $f : X \rightarrow Y$  be an open map of a compactum  $X$  onto a compactum  $Y$  such that all fibers  $f^{-1}(y)$  are homeomorphic to the Cantor set. Does there exist a continuous map  $u : X \rightarrow \mathbb{I}$  such that  $u[f^{-1}(y)] = \mathbb{I}$  for all  $y$  in  $Y$ ? ( $X, Y$  are compact separable metrizable spaces.) A.N. Dranishnikov [102] gave a negative answer. The problem has an affirmative answer in the special cases that  $Y$  is countable-dimensional (V. Gutev [131]) or, more generally, that  $Y$  is a  $C$ -space (M. Levin and J. Rogers [182]).

**Problem 425.** No. W. Olszewski [215] showed that, for each countable limit ordinal  $\alpha$ , there is no universal space for the class of compact separable metrizable spaces with transfinite dimension  $\leq \alpha$ .

#### Eleven annotated problems about continua by H. Cook, W.T. Ingram, A. Lelek

**Problem 429.** No. P. Minc [200] constructed a homeomorphism of a tree-like continuum without a periodic point.

**Problem 432.** Yes. P. Minc [199] constructed an atriodic simple-4-od-like continuum which is not simple-triod-like.

**Problem 433.** Do there exist in the plane two simple closed curves  $J_1$  and  $J_2$  such that  $J_2$  lies in the bounded complementary domain of  $J_1$  but the span of  $J_1$  larger than the span of  $J_2$ ? T. West [287] proved a partial answer: Suppose  $X$  is a continuum which separates the plane and let  $C$  be a convex region contained in a bounded complementary domain of  $X$ ; then  $\sigma(X) \geq \sigma(\partial(C))$ . See also [105].

**Problem 438.** S. Ye and Y.-M. Liu [290] constructed a connected subspace of the plane with infinite span and zero surjective span. This settles Problem 438 in the negative. The problem remains open for continua.

#### Tree-like curves and three classical problems by J.T. Rogers

**Problem 445.** No. P. Minc [201] constructed a hereditarily indecomposable tree-like continuum without the fixed point property.

**Problem 453.** No, see Problem 458.

**Problem 456.** No, see Problem 458.

**Problem 458.** No. J. Prajs [224] constructed a homogeneous, arcwise connected, non-locally connected curve. Since it is arcwise connected, it cannot be mapped onto a solenoid, let alone retracted onto one; this provides a negative solution to Problem 456. The example constructed by Prajs is aposyndetic, but it is not a bundle over the universal curve; this provides a negative solution to Problem 453.

**Problem 460.** P. Krupski and J. Rogers [167] showed that if  $X$  is a homogeneous, finitely cyclic curve that is not tree-like, then  $X$  is a solenoid or  $X$  admits a decomposition into mutually homeomorphic, homogeneous tree-like continua with quotient space a solenoid. This is a solution to Problem 460 for a special case.

**Problem 467.** J. Rogers [231] proved that if  $X$  is a homogeneous, decomposable continuum that is not aposyndetic and has dimension greater than one, then the dimension of its aposyndetic decomposition is one.

**Problem 469.** Yes. S. Solecki [266] showed that no indecomposable continuum has a Borel transversal. See Problem 1079.

**Problem 474.** Yes. Y. Sternfeld and M. Levin [183] showed that for every two-dimensional  $X$  the hyperspace  $C(X)$  (of all subcontinua in  $X$  endowed with the Hausdorff metric) is infinite-dimensional. This was an old problem with many partial results; the conclusion was known if  $\dim X \geq 3$ . M. Levin [181] proved every two-dimensional continuum contains a subcontinuum  $T$  with  $\dim T = 1$  and  $\dim C(T) = \infty$ .

### Problems on topological groups and other homogeneous spaces by W.W. Comfort

**Problem 476.** *Is it a theorem of ZFC that there exist two countably compact groups whose product is not countably compact?* S. Garcia-Ferreira, A.H. Tomita and S. Watson [122] proved that if there is a selective ultrafilter on  $\omega$ , then there are two countably compact groups without nontrivial convergent sequences whose product is not countably compact. It was known that there are examples under MA.

**Problem 477.** *Is there, for every (not necessarily infinite) cardinal number  $\alpha \leq 2^c$ , a topological group  $G$  such that  $G^\gamma$  is countably compact for all cardinals  $\gamma < \alpha$ , but  $G^\alpha$  is not countably compact?* A.H. Tomita [279] showed that  $\alpha = 3$  is such a cardinal, under  $\text{MA}_{\text{countable}}$ . Furthermore, the same author [276] showed that, under  $\text{MA}_{\text{countable}}$ , for every integer  $k > 0$  there exist an integer  $m$  such that  $k \leq m < 2^k$  and a topological group  $G$  such that  $G^m$  is countably compact, and  $G^{m+1}$  is not countably compact.

**Problem 486.** No. See Problem 487.

**Problem 487.** V. Malykhin [121] proved that there is a topological group of countable tightness that is not  $p$ -sequential for any  $p \in \omega^*$ ; this is a negative answer to both Problems 486 and 487(a). P. Gartside, E. Reznichenko and O. Sipacheva [229] proved that there is a homogeneous space of countable tightness that is not  $p$ -sequential for any  $p \in \omega^*$ ; this also is a negative answer to Problem 487(a).

**Problem 497.** No. P. Gartside, E.A. Reznichenko and O.V. Sipacheva [123] showed that there is a Lindelöf topological group with cellularity  $2^{\aleph_0}$ .

**Problem 506.** *Does every infinite compact group contain an infinite Abelian subgroup?* This question was answered positively by E. Zel'manov [294] in 1989.

**Problem 508** (M. Tkachenko). *Can the free Abelian group on  $c$  many generators be given a countably compact group topology?* Under CH, Tkachenko had constructed an example that was even hereditarily separable and connected. A.H. Tomita [278] constructed an example under  $\text{MA}_{\sigma\text{-centred}}$ . P. Koszmider, A.H. Tomita and S. Watson [165] constructed an example without nontrivial convergent sequences, under  $\text{MA}_{\text{countable}}$ . The problem is to construct an example in ZFC.

**Problem 511.** Yes. A. Leiderman, S.A. Morris and V. Pestov [180] gave a complete description of the topological spaces  $X$  such that the free Abelian topological group  $A(X)$  embeds into the free Abelian topological group  $A(\mathbb{I})$  on the closed unit interval. In particular,  $A(X)$  on any embeds into  $A(\mathbb{I})$  for any finite-dimensional compact metrizable space  $X$ .

**Problem 512.** V.V. Uspenskij [284] gave a counterexample to K.H. Hofmann's question.

**Problem 513.** No. F. Javier Trigos-Arrieta [281] proved that no uncountable (Abelian)  $G^\#$  is normal.

**Problem 514.** Yes. D. Shakhmatov [247] proved for every Abelian group  $G$  that the group  $G^\#$  is strongly zero-dimensional.

**Problem 515.** No. This problem of van Douwen was solved by K. Kunen, and by D. Dikranjan and S. Watson. Kunen [171] proved that there are countably infinite Abelian groups whose Bohr topologies are not homeomorphic. Dikranjan and Watson [80] showed that for every cardinal  $\alpha > 2^{2^c}$  there are two groups of cardinality  $\alpha$  with nonhomeomorphic Bohr topologies. Both results are in ZFC. Since discrete spaces of equal cardinality are homeomorphic, these examples also answer Problem 516 in the negative.

**Problem 516.** No. See Problem 515.

**Problem 519.** This problem remains open. A claim to the contrary, broadly circulated in 1991, has since been withdrawn.

**Problem 523** (Wallace Problem). *Is every countably compact topological semigroup with two-sided cancellation a topological group?* D. Robbie and S. Svetlichny [230] found a counterexample under CH. A.H. Tomita [277] produced a counterexample under  $\text{MA}_{\text{countable}}$ .

**Problem 526.** *Is every totally disconnected topological field  $F$  zero-dimensional? What if  $\langle F, T \rangle$  is assumed simply to be a topological ring?* This problem, communicated to Comfort by N. Shell, is also attributed to M.V. Večtomov and V.K. Bel'nov by W. Więśław in his review of [283] by M.I. Ursul; see also [288, p. 254, Problem 21]. Ursul gave a strong negative solution to the problem, with this theorem: For every  $n \in \mathbb{N}$  the topological ring  $\mathbb{R} \times \mathbb{C}^n$  contains a totally disconnected (sub)field of inductive dimension  $n$ .

### Problems in domain theory and topology by J.D. Lawson, M. Mislove

**Problem 532.** Y.-M. Liu and J.-H. Liang [187] proved that a DCPO  $L$  is a continuous  $L$ -domain iff  $[X \rightarrow L]$  is a continuous DCPO for all core compact spaces  $X$ , and in that case,  $[X \rightarrow L]$  is even an  $L$ -domain. This result answers the main part of Problem 532. In a recent paper, J. Lawson and L. Xu [176] have completed the problem by showing that  $L$  is a continuous DCPO in which each principal ideal is a sup-subsemilattice iff  $[X \rightarrow L]$  is a continuous DCPO for all compact and core compact spaces  $X$ .

**Problem 535.** Y.-M. Liu and J.-H. Liang [187,184] proved that a continuous  $L$ -domain  $L$  with a least element is conditionally complete (*bounded complete*) iff  $\text{Is}[X \rightarrow L] = \sigma[X \rightarrow L]$  for all core compact spaces  $X$ . This answers Problem 535 for a special case.

**Problem 540.** *Characterize those topologies that arise as dual topologies. If one iterates the process of taking duals, does the process terminate after finitely many steps with topologies that are duals of each other?* M.M. Kovár [166] showed that for any topological space  $(X, \tau)$ ,  $\tau^{dd} = \tau^{ddd}$ . Further, Kovár classified topological spaces with respect to the number of generated topologies by the process of taking duals. B.S. Burdick [50] had solved this problem for some special cases.

### Problems in the topology of binary digital images by T.Y. Kong, R. Litherland, A. Rosenfeld

The three problems in this section are open.

**Problem 547.** *Find a 3D version of Proposition 2.5: Suppose  $S' \subseteq S$  are finite subsets of  $\mathbb{Z}^2$  and each point in  $S - S'$  is a simple north border point of  $S$  that is 8-adjacent to at least two other points in  $S$ . Then the inclusion of  $S'$  in  $S$  preserves topology.* The original problem did not say how complicated the conditions of the 3D version of Proposition 2.5 were allowed to be. A solution would be of most interest if it was surprisingly simple or if it provided a basis for good thinning algorithms (which need to do much more than

just *preserve topology*). The speed of PCs and the amount of memory they possess have each increased by more than two orders of magnitude since the chapter was written. It now seems feasible to test the correctness of assertions such as possible 3D versions of Proposition 2.5 if they are simple enough on a PC using a combination of brute-force and mathematical results established by C.M. Ma and T.Y. Kong in the mid-1990s.

**On relational denotational and operational semantics for programming languages with recursion and concurrency by J.-J. Ch. Meyer, E.P. de Vink**

The single problem of this section is open.

**Problem 548.** *Does there exist a semantics  $\mathcal{D}'$  for the language Prog which*

- (i) *is compositional, i.e.,  $\mathcal{D}'(d|s_1 * s_2) = \mathcal{D}'(d|s_1) *_{\mathcal{D}'} \mathcal{D}'(D|s_2)$  for every syntactic operator  $*$ ;*
- (ii) *handles recursion by means of fixed point techniques;*
- (iii) *is correct with respect to the operational semantics, i.e.,  $\mathcal{D}' = \mathcal{O}$ ; and*
- (iv) *satisfies  $\mathcal{D}'(d|x) = \mathcal{D}'(d|s)$  for each  $x \Leftarrow s \in d$ .*

**Problems on topological classification of incomplete metric spaces by T. Dobrowolski, J. Mogilski**

**Problem 549.** *Find more absorbing sets.* There have been three approaches to solving this very general and vague problem. There have been many papers and some of the authors are listed here.

- (1) Searching for concrete natural examples of absorbing spaces: J. Baars, T. Banakh, R. Cauty, J. Dijkstra, T. Dobrowolski, H. Gladdines, S. Gul'ko, W. Marciszewski, J. van Mill, J. Mogilski, T. Radul, K. Sakai, T. Yagasaki, M. Zarichnyi.
- (2) Constructing absorbing spaces for certain concrete classes: T. Dobrowolski, J. Mogilski, R. Cauty, M. Zarichnyi, T. Radul, J. Dijkstra.
- (3) General constructions of absorbing spaces:
  - (a) The technique of soft maps and inverse systems: M. Zarichnyi, see [30, §2.3].
  - (b) Producing  $\mathcal{C}$ -absorbing spaces for  $[0, 1]$ -stable classes  $\mathcal{C}$ : T. Banakh, R. Cauty, see [30, §2.4] or [29, §6].

**Problem 551.** M. Zarichnyi [292] constructed absorbing sets for the classes of all separable Borel (projective) sets of  $\dim \leq n$ .

**Problem 552.** M. Zarichnyi [291] constructed absorbing sets for the classes of all separable finite-dimensional sets.

**Problem 554.** Yes, see Problem 900.

**Problem 555.** Yes. A. Chigogidze and M. Zarichnyi announced that every  $n$ -dimensional  $\mathcal{C}$ -absorbing set is representable in  $\mathbb{R}^{2n+1}$ . A proof based on S.M. Ageev's characterization theorem for Nöbeling manifolds [3] is given in [67].

**Problem 558.** In case  $M = \mathbb{R}^\infty$  or  $M = \mathbb{I}^\infty$ , T. Banach and R. Cauty [29] found some quite general conditions on the class  $\mathcal{C}$  under which there exist arbitrarily close to the identity homeomorphisms of  $M$  sending one  $\mathcal{C}$ -absorbing set onto another.

**Problem 560.** No, see Problem 981.

**Problem 561.** T. Banach [26], [30, §5.5.C] proved that the linear hull of the Erdős set in  $\ell^2$  is a noncomplete Borel subspace of  $\ell^2$  which is not a  $\sigma Z$ -space. This example gives negative answers to Problems 561 and 562.

**Problem 562.** No, See Problem 561.

**Problem 564.** No. R. Cauty observed that the open unit ball in  $\ell_2$  enlarged by a subset of the unit sphere that is of a suitable higher Borel complexity yields a (nonclosed) convex set that provides a negative answer to Problems 564 and 565. A positive answer to Problem 564 is found for a wide class of  $\lambda$ -convex sets (including topological groups and closed convex sets in locally convex linear metric separable spaces) and classes  $\mathcal{C}$  (including almost all absolute Borel and projective classes); see [27], [28, §4.2, §5.3], [30].

**Problem 565.** See Problem 564.

**Problem 566.** No, see Problem 981. Cauty's example is a complete metric linear space without the admissibility property. As observed in [81, p. 764] (see also [39, Proposition 3.3]) such an example fails to have the homeomorphism extension property between compacta.

**Problem 567.** Find an infinite-dimensional absorbing set in  $\mathbb{R}^\infty$  which does not admit a group structure. Such an absorbing set was constructed by M. Zarichnyi [30, §4.2.D]. This set admits no cancellable continuous operation  $X \times X \rightarrow X$  and thus is not homeomorphic to any convex subset of a linear topological space. A  $\sigma$ -compact absorbing space with the same properties was constructed by T. Banach and R. Cauty [29].

**Problem 568.** T. Dobrowolski and J. Mogilski note that the notion of  $\lambda$ -convexity used in their set of problems is stronger than the usual one. In particular, their  $\lambda$ -convex sets are subsets of metric topological groups. Usually, a set is meant to be  $\lambda$ -convex if it admits an equiconnecting function. The absolute retract property implies the usual notion of  $\lambda$ -convexity. So, having in mind such a weaker notion, the assumption  $\lambda$ -convex would be superfluous in Problems 560, 561, 563, 564, 565, and 568. With such a weaker notion of  $\lambda$ -convexity, the examples in Problem 567 provide a negative answer to Problem 568.

**Problem 569.** By modifying a counterexample due to W. Marciszewski [189], T. Banach constructed a linear absorbing subset of  $\mathbb{R}^\infty$  that is not homeomorphic to any convex subset of a Banach space as well as a linear absorbing subset of  $\ell^1$  that is not homeomorphic to any convex subset of a reflexive Banach space. See [30, §5.5.B]. These provide negative answers to Problems 569 and 570.

**Problem 570.** No, see Problem 569.

**Problem 575.** T.N. Nguyen, J.M.R. Sanjurjo and V.A. Tran [211] proved that Roberts' example is an AR, therefore homeomorphic to the Hilbert cube. This provides a positive answer to Problem 575.

**Problem 576.** T. Banach [27] gave a positive answer for a special case where  $W$  is a subset of a locally convex space and  $W$  contains an almost internal point (the latter occurs if  $W$  is centrally symmetric).

**Problem 584.** R. Cauty [52] constructed a family of continuum-many topologically distinct  $\sigma$ -compact countably dimensional pre-Hilbert spaces. This family provides answers to Problems 584 and 585.

**Problem 585.** No, see Problem 584.

**Problem 587.** C. Bessaga and T. Dobrowolski [38] proved that every  $\sigma$ -compact locally convex metric linear space is homeomorphic to a pre-Hilbert space and so can be densely embedded into  $\ell_2$ .

**Problem 588.** Find interesting (different from  $\Sigma$  and from that of [84, Ex. 4.4])  $\sigma$ -compact absorbing sets which are not countable-dimensional. For every countable ordinal  $\alpha$ , T. Radul [228] constructed a  $C$ -compactum universal for the class of all compacta with  $\dim_C X \leq \alpha$ . Using this result, Radul proved that for uncountable many ordinals  $\beta$  there exist non-countable-dimensional pre-Hilbert spaces  $D_\beta$  which are absorbing spaces for the class of compacta with  $\dim_C$  less than  $\beta$ . Here,  $\dim_C$  is Borst's transfinite extension of covering dimension which classifies  $C$ -compacta. M. Zarichnyi [293] showed that some absorbing sets for classes of compacta of given cohomological dimension are not countable-dimensional.

**Problem 590.** R. Cauty [53, Theorem 3] answered this problem affirmatively. T. Banach obtained also a result in this direction.

**Problem 591.** R. Cauty [56] proved that given any Banach space  $E$ , any linearly independent Cantor set  $C$  in  $E$  and any subset  $A$  of  $C$ , then the power  $(\text{span}(A))^\infty$  is not universal for the additive class  $\mathcal{A}_2$ . This provides negative answers to Problems 591 and 592.

**Problem 592.** No, see Problem 591.

**Problem 593.** No, R. Cauty [51] showed that there are continuum many topologically different such spaces. R. Cauty and T. Dobrowolski [60] proved that there are at least uncountably many examples.

**Problem 594.** T. Banach and R. Cauty gave a positive answer. See [30, Theorem 4.2.3].

**Problem 596.** R. Cauty, T. Dobrowolski and W. Marciszewski [61] gave a positive answer.

**Problem 597.** This important problem was solved in the negative. R. Cauty [57] proved that for every countable ordinal  $\alpha \geq 3$  there exist countable completely regular spaces  $X_\alpha$  and  $Y_\alpha$  such that the spaces  $C_p(X_\alpha)$  and  $C_p(Y_\alpha)$  are Borelian of class exactly  $\mathcal{M}_\alpha$  but are not homeomorphic. This provides negative answers to Problems 597, 598, 599 and 602.

**Problem 598.** No, see Problem 597.

**Problem 599.** No, see Problem 597.

**Problem 600.** No. The space  $c_{F_0}$  is contained in a  $\sigma$ -compact subset of  $\mathbb{R}^\infty$ , while  $(\mathbb{R}_f^\infty)^\infty$  is not contained in a  $\sigma$ -compact subset of  $(\mathbb{R}^\infty)^\infty$ . This observation was made by many authors.

**Problem 601.** This problem was partially solved by R. Cauty, T. Dobrowolski and W. Marciszewski. The answer to the first part of the question is yes. The answer to the second part is yes if one additionally assumes that the space is a countable union of  $Z$ -sets. See [61].

**Problem 602.** No, see Problem 597.

**Problem 603.** T. Dobrowolski and W. Marciszewski [83] proved that in every infinite-dimensional Fréchet space  $X$ , there is a linear subspace  $E$  such that  $E$  is a  $F_{\sigma\delta\sigma}$  subset of  $X$  and contains a retract  $R$  so that  $R \times E^\omega$  is not homeomorphic to  $E^\omega$ . This provides negative answers to Problems 603, 605 and 606.

**Problem 604.** T. Banach gave an example of a Borel pre-Hilbert space  $E$  homeomorphic to  $E \times E$  but not to  $E_f^\infty$ . See [26] and [30, §5.5.C].

**Problem 605.** No, see Problem 603.

**Problem 606.** No, see Problem 603.

**Problem 608.** The Nöbeling spaces  $N_k^{2k+1}$  are the  $k$ -dimensional analogues of Hilbert space.  $N_k^{2k+1}$  is a separable, topologically complete (i.e., Polish)  $k$ -dimensional absolute extensor in dimension  $k$  (i.e.,  $AE(k)$ ) with the property that any map of an at most  $k$ -dimensional Polish space into  $N_k^{2k+1}$  can be arbitrarily closely approximated by a closed embedding (i.e., it is a strong  $k$ -universal Polish space). Problem 608 asks if these



properties characterize the Nöbeling spaces  $N_k^{2k+1}$ , for  $k \geq 1$ . S.M. Ageev [3] announced that these properties characterize the Nöbeling spaces  $N_k^{2k+1}$ , for every  $k \geq 2$ . The one-dimensional case was proved by K. Kawamura, M. Levin and E.D. Tymchatyn [155].

### Problems about finite-dimensional manifolds by R.J. Daverman

**Problem 614.** See Problem 615.

**Problem 615.** G. Perelman [222] announced a proof of Thurston's Geometrization Conjecture and, consequently, the Poincaré Conjecture. This remarkable work is still being scrutinized.

**Problem 620.** See Problem 615.

**Problem 625.** D. Halverson [134] found new conditions on  $X$  under which  $X \times \mathbb{R}$  is a manifold.

**Problem 626.** J. Bryant, S. Ferry, W. Mio and S. Weinberger [47,48] gave a negative answer to the Resolution Problem. They constructed compact ANR homology manifolds in dimensions  $\geq 5$  which do not admit a resolution. It is still not known if every generalized 4-manifold admits a cell-like resolution.

**Problem 639.** The class of closed manifolds that cover themselves does not coincide with the class of closed manifolds that cover themselves both regularly and cyclically.

**Problem 642.** Does  $\text{cat}(M \times S^r) = \text{cat}(M) + 1$ ? This conjecture of Ganea refers to the Lusternik–Schnirelman category  $\text{cat}(P)$  of a space  $P$ . N. Iwase [138] has shown that, in general, the answer is negative.

**Problem 643.** If  $M$  is a closed PL manifold, does  $\text{cat}(M - \text{point}) = \text{cat}(M) - 1$ ? N. Iwase [139] also has given a negative answer to this question.

**Problem 644.** L. Montejano [202] gave a positive answer to Problem 68 from *The Scottish Book*, as was noted in a footnote in the book. If  $M$  is a compact subset of  $\mathbb{R}^n$  and for some  $k$  with  $1 \leq k \leq n$  every  $k$ -dimensional hyperplane  $H$  from  $\mathbb{R}^n$  that intersects  $M$  has  $H \cap M$  acyclic (Čech cohomology, integer coefficients), then  $M$  is convex.

**Problem 645.** Borsuk's Problem was answered in the negative by J. Kahn and G. Kalai [144].

**Problem 649.** J. Dydak and J. Walsh [113] have shown the existence of a cell-like, dimension raising map on a 2-dimensional compactum. As a result, maps on  $\mathbb{R}^n$  can raise dimension when  $n = 5, 6$ . Whether the same is true for  $n = 4$  remains unsettled.

**Problem 655.** The examples of [47,48] give a negative solution to Thickstun's Full Blow-up Conjecture.

**Problem 670.** A. Chigogidze [65] proved that if  $X$  and  $Y$  are shape equivalent  $LC^k$  compacta, then they are  $UV^k$ -equivalent.

**Problem 677.** U.H. Karimov and D. Repovš [147] showed that there exists noncontractible cell-like compactum whose suspension is contractible. Their example is 3-dimensional, so they asked whether there exist 1- or 2-dimensional counterexamples.

**Problem 685.** R. Daverman and F. Tinsley [74] gave a negative answer.

**Problem 686.** The broader, initial question was answered in the affirmative by R. Daverman and F. Tinsley [73]

**Problem 690.** *For which  $n$ -manifolds  $N$  and integers  $k$  does the hypothesis that all elements of  $G$  are copies of  $N$  imply  $p: M \rightarrow B$  is an approximate fibration?* R. Daverman has counterexamples to the conjectures that  $\pi_1(N)$  is finite or  $k = 2$  suffice.

**Problem 691.** This problem was solved by R. Daverman [71].

**Problem 700.** C. Guilbault [130] discovered examples of compact, contractible  $n$ -manifolds,  $n \geq 9$ , other than the  $n$ -cell, which have disjoint spines. M. Sanders [244] refined Guilbault's work to get examples for  $n \geq 5$ . However, neither work addresses the possibility of disjoint spines in contractible 4-manifolds.

**Problem 704.** *Which homology  $n$ -spheres  $K$  bound acyclic  $(n+1)$ -manifolds  $N$  such that  $\pi_1(K) \rightarrow \pi_1(N)$  is an isomorphism? Is there a homology 3-sphere example?* R. Daverman [72] gave a negative answer to the latter problem.

**Problem 708.** *Can every  $S^n$ -like continuum can be embedded in  $\mathbb{R}^{2n}$ ?* A metric space  $X$  is said to be  $S^n$ -like if there exist  $\varepsilon$ -maps  $X \rightarrow S^n$  for every  $\varepsilon > 0$ . P.M. Akhmet'ev [6] proved that this is true for  $n \neq 1, 2, 3, 7$ . See also [8] for an easier solution. Akhmet'ev [7] also proved that for every  $k$ , there exists  $n$  such that every  $S^n$ -like continuum can be embedded in  $\mathbb{R}^{2n-k}$ .

**Problem 710.** T. Rushing and R. Sher [235] showed that there is a cellular wedge  $A \vee B$  in  $\mathbb{R}^3$  where one of the constituent parts is not cellular.

#### A list of open problems in shape theory by J. Dydak, J. Segal

**Problem 711.** L. Rubin [232] proved a sum formula for integral cohomological dimension: If  $X = A \cup B$  is metrizable and the integral cohomological dimensions of the summands are finite then  $\dim_{\mathbb{Z}} X \leq \dim_{\mathbb{Z}} A + \dim_{\mathbb{Z}} B + 1$ . V.I. Kuz'minov had asked if the same

estimate holds for the cohomological dimension over any group  $G$ . Counterexamples for the general problem were given by A.N. Dranishnikov, D. Repovš and E.V. Ščepin [103] and by J. Dydak [107]. The full solution of a more general problem is contained in [108].

**Problem 712.** See Problem 649.

**Problem 714.** See Problem 649.

**Problem 719.** P. Mrozik [209] showed that for a finite dimensional compactum  $X$  the difference between the minimum dimension of compacta shape equivalent to  $X$  and the minimum dimension of compacta CE-equivalent to  $X$  can be arbitrarily large.

**Problem 721** (Borsuk). *Is a movable continuum pointed movable?* In [111], it is shown that approximate polyhedra with the fixed point property are pointed movable. In particular, approximate polyhedra with Euler characteristic of the one-point space are pointed movable. I. Ivanišić and L. Rubin [137] proved that projective movable continua are pointed movable.

**Problem 741.** *Is every shape equivalence a strong shape equivalence?* J. Dydak and S. Nowak [110] showed that  $f : X \rightarrow Y$  is a strong shape equivalence of compacta if and only if  $f \times \text{id}_Q : X \times Q \rightarrow Y \times Q$  is a shape equivalence for each CW complex  $Q$ .

### Problems on algebraic topology by G.E. Carlsson

**Problem 754** (A. Adem). *Let  $G$  be a finite  $p$ -group. If  $H^n(G; \mathbb{Z})$  has an element of order  $p^r$  for some value of  $n$ , does the same follow for infinitely many  $n$ ?* No. J. Pakianathan [217] constructed a counterexample.

**Problem 757** (M. Feshbach). *Can one give a useful alternative description of the necessary  $p$ -groups?* A. Adem and D. Karagueuzian [1] proved that a finite group  $G$  has a Cohen-Macaulay mod  $p$  cohomology ring with non-detectable elements if and only if  $G$  is a  $p$ -group such that every element of order  $p$  in  $G$  is central. In [236], D. Rusin gave an example of a 2-group which has undetectable cohomology although it contains non-central elements of order 2, thus also disproving the conjecture for  $p = 2$ . (Note: non-detectable elements are now called essential cohomology classes.)

**Problem 766.** J. Klippenstein and V. Snaith [157] proved a conjecture of Barratt–Jones–Mahowald concerning framed manifolds having Kervaire invariant one.

**Problem 788.** *Prove a 14-connected finite  $H$ -space is acyclic.* This is resolved using some results of A. Jeanneret and the paper by J.P. Lin [185].

**Problem 790.** *Are there any finite loop spaces whose mod 2 cohomology is not the mod 2 cohomology of a Lie group?* Yes. W.G. Dwyer and C.W. Wilkerson [106] constructed a

$\mathbb{F}_2$ -complete loop space  $\Omega B$  with  $H^*(\Omega B; \mathbb{F}_2) \cong \mathbb{F}_2 \frac{[x_7]}{x_7^4} \otimes \Lambda(y_{11}, z_{13})$  This cohomology is not the mod 2 cohomology of any Lie group. This result also answers Problem 794.

**Problem 794.** *Is a 6-connected finite  $H$ -space a product of seven spheres?* No, see Problem 790.

**Problem 795.** *If  $X$  is a finite loop space is  $H^*(X; \mathbb{Z}) = H^*(\text{Lie group}; \mathbb{Z})$ ?* The mod 3 cohomology rings of a finite  $H$ -space have been classified. This gives partial information to Problem 795. For any finite  $H$ -space  $X$  with  $H_*(X; \mathbb{Z}_3)$  associative the cohomology algebra  $H^*(X; \mathbb{Z}_3)$  is isomorphic to the cohomology algebra of a finite product of  $E_8$ s,  $X(3)$ s and odd dimensional spheres. Here  $X(3)$  is the finite  $H$ -space constructed by J.R. Harper. See [186].

**Problem 809** (W. Browder). *If  $G = (\mathbb{Z}/p)^k$ ,  $p$  prime, acts freely on  $\mathbb{S}^{n_1} \times \mathbb{S}^{n_2} \times \cdots \times \mathbb{S}^{n_r}$ , is  $k \leq r$ ?* In his thesis, E. Yalçın [289] proved this conjecture in the case when the spheres are all circles.

**Problems 810–813** (M. Morimoto). *Do there exist smooth, one fixed point actions of compact Lie groups (possibly finite groups) on  $S^3$ ,  $D^4$ ,  $S^5$ , or  $S^8$ ?* When  $G$  is a compact Lie group, if a  $G$ -manifold has exactly one  $G$ -fixed point then the action is said to be a *one fixed point action*. M. Furuta proved that there are no smooth one fixed point actions on  $S^3$  of finite groups. This was also proved by N.P. Buchdahl, S. Kwasik and R. Schultz [49, Theorem I.1]. In [49, Theorem II.2], they proved that there are no locally linear, one fixed point actions on homology 4-dimensional spheres of finite groups; in [49, Theorem II.4], this was proved for homology 5-dimensional spheres of finite groups. Morimoto [204,205] proved that there exist smooth one fixed point actions on  $S^6$  of  $A_5$ . A. Bak and Morimoto [13, Theorem 7] proved that there exist smooth one fixed point actions on  $S^7$  of  $A_5$ . Bak and Morimoto [14] proved that there are smooth one fixed point actions on  $S^8$  of  $A_5$ .

**Problems 822–824.** These problems concern the question of which smooth manifolds can occur as the fixed point sets of smooth actions of a given compact Lie group  $G$  on disks (respectively, Euclidean spaces). In the case where  $G$  is a finite group not of prime power order, complete answers go back to B. Oliver [214]. Specifically, for a compact smooth manifold  $F$  (respectively, a smooth manifold  $F$  without boundary), Oliver described necessary and sufficient conditions for  $F$  to occur as the fixed point set of a smooth action of  $G$  on a disk (respectively, Euclidean space). Oliver's description of the necessary and sufficient conditions imply affirmative answers to Problems 822, 823 and 824. In the case where  $G$  is of  $p$ -power order for a prime  $p$ , a compact smooth manifold  $F$  occurs as the fixed point set of a smooth action of  $G$  on a disk if and only if  $F$  is mod  $p$ -acyclic and stably complex. This follows from Smith theory and the results of L. Jones [140]. A similar result holds in the case where  $G$  is a compact Lie group such that the identity connected component  $G_0$  of  $G$  is Abelian (i.e.,  $G_0$  is a torus) and  $G/G_0$  is a finite  $p$ -group for a prime  $p$ . Moreover, K. Pawłowski [219] proved that for such a group  $G$ , a smooth manifold  $F$  without boundary occurs as the fixed point set of a smooth action of  $G$  on some Euclidean space if and only if  $F$  is mod  $p$ -acyclic and stably complex. The article

of K. Pawałowski [220] gives an up-to-date survey of results and answers to the question of which smooth manifolds can occur as the fixed point sets of smooth actions of a given compact Lie group  $G$  on Euclidean spaces, disks, and spheres.

### Problems in knot theory by L.H. Kauffman

**Problem 843.** G. Spencer-Brown gave a counterexample to the Switching Conjecture. See the second status report [194] for a diagram.

**Problem 844.** This problem asks the reader to understand G. Spencer-Brown's proof of the four color theorem using the diagrammatic technique of formations. See the articles by L.H. Kauffman [153,154] for more information on this approach.

**Problem 845.** *Suppose that  $K\#$  is alternating, reduced and achiral. Then there exists an alternating diagram  $K$ , ambient isotopic to  $K\#$ , such that the graphs  $G(K)$  and  $G^*(K)$  are isomorphic graphs on the two-dimensional sphere.* This is a corrected version of the conjecture that appeared in the book. O.T. Dasbach and S. Hougardy [70] gave a counterexample to this conjecture.

### Open problems in infinite dimensional topology by J.E. West

**Problem 870.** J. Dydak and J.J. Walsh [112] proved that for each  $n \geq 4$ , there is a metrizable space  $X$  with  $\dim_{\mathbb{Z}} X \leq n$  and  $\dim_{\mathbb{Z}} Y \geq n + 1$  for every Hausdorff compactification  $Y$  of  $X$ . This is a negative answer to the second part of Problem 870.

**Problem 871.** A. Chigogidze [66] has shown that for each  $n \geq 0$  there exists a completely metrizable space  $X_n$  such that the following conditions are equivalent for each metrizable compactum  $K$ :

- (a) the integral cohomological dimension of  $K$  is less than or equal to  $n$ , and
- (b)  $K$  admits an embedding into  $X_n$ .

Since there exist separable metrizable spaces with finite cohomological dimension having no metrizable compactification with the same finite cohomological dimension (see Problem 870), it is unclear whether this (interesting) result is a partial answer to this problem. J. Dydak and J. Mogilski [109] found a proof of the existence of a universal space for the class of separable metric spaces of cohomological dimension at most  $n$ . This improves Chigogidze's result.

**Problem 885.** This is the same as Problem 670.

**Problem 890.** No, see Problem 981.

**Problem 891.** R. Cauty [55] proved that a metric space is an ANR if and only if every open subset of  $X$  has the homotopy type of a CW-complex. K. Sakai [238] gave an alternative proof.

**Problem 892.** No, for the first part. See Problem 981.

**Problem 894.** No, for both parts. See Problem 981.

**Problem 899.** *Are the Banach–Mazur compacta  $BM(n)$  absolute retracts? Are they Hilbert cubes?* The Banach–Mazur compactum  $BM(n)$  is the set of isometry classes of  $n$ -dimensional Banach spaces appropriately topologized. The Banach–Mazur compacta are now known to be absolute retracts: the key ingredient for this result is due to P. Fabel. In the case  $n = 2$ ,  $BM(2)$  is not a Hilbert cube: S. Antonyan [9] proved that  $BM_0(2)$  (see below) is not contractible and  $BM(2)$  is not homogeneous. S.M. Ageev and D. Repovš [5] proved that  $BM_0(2)$  is a Hilbert cube manifold. Furthermore, Antonyan [10] proved that for every  $n \geq 2$ ,  $BM_0(n)$  is a  $[0, 1)$ -stable Hilbert cube manifold. S.M. Ageev, S.A. Bogatyř and D. Repovš [4] showed that for every  $n \geq 2$ ,  $BM_0(n)$  is a Hilbert cube manifold. Here  $BM(n) = L(n)/O(n)$  is modeled (see, e.g., [9]) as the orbit space of an action of the orthogonal group  $O(n)$  on the hyperspace  $L(n)$  of all centrally symmetric, compact, convex bodies  $A$  in  $\mathbb{R}^n$  for which the Euclidean unit ball is the minimal volume ellipsoid containing  $A$ .  $BM_0(n)$  is the complement of the unique singular (Euclidean) point in  $BM(n)$ . More generally, Antonyan [10] described new topological models for  $BM(n)$  and proved that for any closed subgroup  $H \subset O(n)$ , the orbit space  $L_0(n)/H$  is a  $[0, 1)$ -stable Hilbert cube manifold. ( $L_0(n) \subset L(n)$  is the complement of the unique  $O(n)$ -fixed point.) Antonyan also proved that for any closed subgroup  $K \subset O(n)$  acting non-transitively on  $\mathbb{S}^{n-1}$ , the  $k$ -orbit space  $L(n)/K$  and the  $K$ -fixed point set  $L(n)/K$  are Hilbert cubes. (In particular,  $L(n)$  is a Hilbert cube.)

**Problem 900.** T. Banach gave positive answers to both parts of this problem. For part (a), see [30, Theorem 1.3.2]. This result was reproved afterward by T. Dobrowolski [82]. For part (b), see [25], which also contains the positive answer to the nonseparable version of part (a). This implies a positive answer to Problem 554 since each  $\mathcal{C}$ -absorbing space is an AR with SDAP.

**Problem 901.** *Is every  $\sigma$ -compact space with the compact extension property an ANR?* T.N. Nguyen and K. Sakai [210] proved that a  $\sigma$ -compact space is an AR (ANR) iff it is equi-connected (locally equi-connected) and has the compact neighbourhood extension property.

**Problem 933.** This problem was solved by J. van Mill and J.E. West [191]: every compact Lie group admits semifree actions  $\alpha$  and  $\beta$  on  $Q$  such that their fixed point sets are identical, their orbit spaces are homeomorphic to  $Q$ , but  $\alpha$  and  $\beta$  are not conjugate.

**Problem 941.** Problems 941 and 942 ask to construct the foundations of equivariant  $Q$ -manifold theory for arbitrary compact groups. S.M. Ageev [2] established equivariant

versions of the basic theorems on  $Q$ -manifolds and then proved a characterization theorem which is an extension of the Toruńczyk  $Q$ -manifold characterization theorem to the equivariant case. Earlier results are due to Steinberger and West (for arbitrary finite groups).

**Problem 942.** See Problem 941.

**Problem 968.** K. Sakai and R. Wong [239] proved that  $LIP(X, Y)$  is a  $\Sigma$ -manifold in case  $X$  is a non-totally disconnected compact metric space and  $Y$  is a separable finite-dimensional locally compact absolute  $LIP$  neighborhood extensor with no isolated points.

**Problem 981.** R. Cauty [54] constructed a metrizable  $\sigma$ -compact linear topological space that is not admissible, and hence not an absolute retract, and such that it can be embedded as a closed linear subspace into an absolute retract. This important result answers in the negative Problems 560, 566, 890, the first part of Problems 892, 894, 981, 982, 984(b), 985 except for the case of compact spaces, the first part of Problems 988 and 995.

**Problem 982.** No, see Problem 981.

**Problem 984.** No to part (b), see Problem 981.

**Problem 985.** See Problem 981.

**Problem 986.** R. Cauty [59] proved that Schauder's fixed point theorem holds without the assumption of local convexity: every compact convex subset of a linear metric space has the fixed point property.

**Problem 988.** No, to both parts. See Problems 981 and 996.

**Problem 989.** No, see Problem 996.

**Problem 990.** P.V. Semenov [246] gave a negative answer to this problem.

**Problem 993.** *Is every locally connected closed additive subgroup of a Hilbert space an ANR?* R. Cauty [58] gave a negative answer. J. Grabowski [124] obtained a very elegant and short solution.

**Problem 995.** No, see Problem 981.

**Problem 996.** W. Marciszewski [189] constructed two counterexamples.

- (1) There exists a separable normed space  $X$  such that  $X$  is not homeomorphic to any convex subset of a Hilbert space. In particular,  $X$  is not homeomorphic to a pre-Hilbert space.
- (2) There exists a separable locally convex linear metric space  $Y$  such that  $Y$  is not homeomorphic to any convex subset of a normed space. In particular,  $Y$  is not homeomorphic to a normed space.

The first example gives negative answers to Problems 988 and 989. The second example gives a negative answer to Problem 996.

**Problem 1008.** T. Banach and K. Sakai [237,31] answered these and many other questions about manifolds modeled on  $\mathbb{R}^\infty$  and  $\sigma$ .

**Problem 1022.** Let  $G$  be a compact Lie group. Let  $\exp G$  be the hyperspace of all nonempty compact subsets with the Hausdorff metric. What is the structure of the orbit space  $(\exp G)/G$ ? S. Antonyan [10, Theorem 1.1] proved that if  $G$  be a compact Lie group and  $X$  is a metrizable  $G$ -space then the following are equivalent:

- (1)  $X$  is locally continuum-connected (respectively, connected and locally continuum-connected);
- (2)  $\exp X$  is a  $G$ -ANR (respectively, a  $G$ -AR);
- (3)  $(\exp X)/G$  is an ANR (respectively, an AR).

This result gives some information about the orbit space (specifically, it is an ANR, and even an AR if  $G$  is connected). Also, it follows that if  $G$  is a finite group acting on a nondegenerate Peano continuum  $X$ , then the orbit space  $(\exp X)/G$  is a Hilbert cube.

#### Problems in $C_p$ -theory by A.V. Arhangel'skiĭ

**Problem 1026.** W. Marciszewski [188] proved that there exists an infinite compact space  $X$  such that there is no continuous linear surjection from the function space  $C_p(X)$  onto  $C_p(X) \times \mathbb{R}$ . In particular, the space  $C_p(X)$  is not linearly homeomorphic to the product  $C_p(X) \times E$  for any nontrivial linear topological space  $E$ . This result also provides negative answers to Problems 1050 and 1051 in the case of linear homeomorphisms.

**Problem 1028.** Let  $X$  be an infinite compact space. Is it true that the Banach spaces  $C_B(X)$  and  $C_B(X) \times \mathbb{R}$  are linearly isomorphic? P. Koszmider [163] proved that there is an infinite compact space  $X$  such that the Banach space  $C_B(K)$  of continuous functions on  $K$  is not isomorphic to its hyperplanes (one-codimensional subspaces). For such a space one necessarily has that  $C_B(K)$  is not isomorphic to  $C_B(K + 1)$  or to  $C_B(K) \times \mathbb{R}$  which answers Problem 1028 in the negative. The space has much stronger properties:  $C_B(K)$  is not isomorphic to any of its proper subspaces nor to any of its proper quotients.

**Problem 1029.** No. R. Pol [223] constructed several examples of metrizable spaces  $X$  with  $X$  not 1-equivalent to  $X \oplus X$ . Some  $X$  were even compact. The case of homeomorphisms remains widely open.

**Problem 1040.** See Problem 1041.

**Problem 1041.** Is it true that every space  $Y$  of countable tightness is homeomorphic to a subspace (to a closed subspace) of  $C_p(X)$  where  $X$  is Lindelöf? (where the tightness



of  $C_p(X)$  is countable)? M. Sakai [240] proved that every separable, compact, linearly ordered topological space is second countable if it is homeomorphic to a subspace of  $C_p(X)$  where  $X$  is Lindelöf. This result can be applied to obtain negative answers to Problems 1040 and 1041.

**Problem 1046.** A. Leiderman, S.A. Morris and V. Pestov [180, Theorem 4.4] proved that for every finite-dimensional metrizable compactum  $X$ , the free locally convex space  $L(X)$  embeds into  $L([0, 1])$  as a locally convex subspace (equivalently, there exists a linear continuous surjection  $L : C_p[0, 1] \rightarrow C_p(X)$ ).

**Problem 1047.** A. Leiderman, M. Levin and V. Pestov [178, 179] proved that for every finite dimensional compactum  $Y$  there exists a 2-dimensional compactum  $X$  that admits a linear continuous open surjection from  $C_p(X)$  onto  $C_p(Y)$ .

**Problem 1050.** See Problem 1026.

**Problem 1051.** See Problem 1026.

**Problem 1052.** G.A. Sokolov [265] solved this problem of S.P. Gul'ko by constructing a compact space  $X$  whose iterated continuous function spaces  $C_p(X)$ ,  $C_p(C_p(X))$ ,  $\dots$  are Lindelöf, but  $X$  is not a Corson compactum.

**Problem 1053.** This is equivalent to the following question: *Suppose  $X$  is a compact Hausdorff space. If there exists a Lindelöf subspace  $Y$  of  $C_p(K)$  that separates points of  $X$ , must  $X$  be countably tight?* There are consistent negative and positive answers to this question. O. Pavlov [218] proved that, assuming  $\diamond$ , there exists a compact Hausdorff space  $X$  of uncountable tightness such that  $C_p(X)$  contains a separating family which is a Lindelöf space. Arhangel'skiĭ [11] had shown that a positive answer is consistent.

**Problem 1057.** O. Okunev [213] proved that if there is an open mapping from a subspace of  $C_p(X)$  onto  $C_p(Y)$ , then  $Y$  is a countable union of images of closed subspaces of finite powers of  $X$  under finite-valued upper semicontinuous mappings. Consequently, if  $X$  and  $Y$  are  $t$ -equivalent compact spaces, then  $X$  and  $Y$  have the same tightness.

**Problem 1059.** M. Sakai [241] described a realcompact non-Lindelöf space  $X$  such that  $C_p(X)$  is Lindelöf.

**Problem 1060.** As noted in the book, this problem was answered in [12]: If  $X$  and  $Y$  are metrizable spaces and there is a continuous linear surjection from  $C_p(X)$  onto  $C_p(Y)$ , then  $Y$  is completely metrizable whenever  $X$  is.

### Problems in topology arising in analysis by R.D. Mauldin

**Problem 1068.** *Is there a Borel set  $M$  in  $\mathbb{R}$  which meets every straight line in exactly two points? Can  $M$  be a  $G_\delta$  set?* These questions are still unsolved. J.J. Dijkstra and J. van Mill [79] showed that if a two-point set is  $G_\delta$  then it must be nowhere dense in the plane.

**Problem 1069.** *Must a two-point set always be zero-dimensional?* J. Kulesza [168] showed that any subset of  $\mathbb{R}^2$  which intersects no line in more than two points either is zero-dimensional or else contains an arc. An  $n$ -point set is a subset of the plane which meets every line in exactly  $n$  points. See [170] for a generalization of this result to  $n$ -point sets.

**Problem 1070.** *Can a compact zero-dimensional partial two-point set always be extended to a two-point set?* A negative answer for this corrected problem has been obtained independently by R. Dougherty, by J.J. Dijkstra and J. van Mill [78], and by D. Mauldin [190].

**Problem 1071.** D. Boyd and D. Mauldin [41] characterized the order type of  $S$ , the set of all Pisot–Vijayaraghavan numbers, as follows. First let  $a_1 = \omega + 1 + \omega^*$  and for each positive integer  $n$ , set  $a_{n+1} = (a_n \cdot \omega) + 1 + (a_n \cdot \omega)^*$ . Then the order type of  $S$  is the ordered sum  $\sum_{n=1}^{\infty} a_n$ .

**Problem 1072.** The Cantor–Bendixson derived set order of  $S$ , the set of all Pisot–Vijayaraghavan numbers, is  $\omega$ . This follows from a theorem of Duresnoy and Pisot who showed that the minimal element of  $S^{(n)} > n^{1/4}$ . The best result concerning upper bounds of  $\min S^{(n)}$  seems to be one of M.J. Bertin [37]. She showed that  $k \in S^{(2k-2)}$ , for  $k > 1$ . For further comments and an interesting conjecture of Boyd concerning  $\min S^{(n)}$ , see [40].

**Problem 1073.** A. Krawczyk solved this problem. The countable set of finite shift maximal sequences on the parity-lexicographic order is order-isomorphic to  $(\mathbb{Q}^+ \cup \{0\}) \times \omega$  with lexicographic ordering.

**Problem 1074.** H. Becker and R. Dougherty [32] showed that there do not exist uncountably many Borel selectors on  $\mathcal{K}(\mathbb{I})$ , the hyperspace of all compact subsets of the unit interval, whose ranks (as Borel functions) are bounded below  $\omega_1$ . Thus, there are continuum many such selectors if and only if CH holds.

**Problem 1075.** This problem is misstated. G. Debs and J. Saint Raymond [75] answered a simpler version of this problem. They showed that there is a Borel set  $B$  in the unit square with all vertical and horizontal fibers co-meager.

**Problem 1075.** *Let  $B$  be a Borel set of  $[0, 1] \times [0, 1]$  such that each horizontal and each vertical fiber of  $B$  is a dense  $G_\delta$ -set. Can  $B$  be filled up by a collection of pairwise disjoint graphs of Borel isomorphisms of  $[0, 1]$  onto  $[0, 1]$ ?*

This corrected problem is unsolved. Debs and Saint Raymond also showed that such a set  $B$  does contain a Borel matching—the graph of some Borel isomorphism. From this we get that  $B$  does contain  $\aleph_1$  Borel matchings.

**Problem 1076.** This problem has been partially solved by F. Ledrappier [177]. He showed that if  $f(x) = \sum_{p=0}^{\infty} 2^{p(s-2)} A(2^p x)$ , where  $A$  is piecewise  $C^{1+\varepsilon}$  and  $1 \leq s < 2$ , then the dimension of  $f$  is  $s$ , provided  $s$  is an Erdős number.

**Problem 1079.** *Is there a Borel subset of an indecomposable continuum which meets each equivalence class (decomposed into composants) in exactly one point?* S. Solecki [266] showed that up to Borel bireducibility this equivalence relation can only be of two types. Consequently, the answer is negative.

### Continuum theory and topological dynamics by M. Barge, J. Kennedy

**Problem 1080.** *Is there a Hénon map that is transitive on some continuum?* M. Benedicks and L. Carleson [34] proved that there is a positive Lebesgue measure set of parameters for which the Hénon map has a transitive attractor.

**Problem 1084.** *Let  $\{p_1, p_2, \dots, p_n\}$  be a set of  $n \geq 2$  distinct points in the sphere  $S^2$ . Is there a homeomorphism of  $S^2 \setminus \{p_1, p_2, \dots, p_n\}$  such that every orbit of the homeomorphism is dense?* See Problem 1085.

**Problem 1085.** *Is there a homeomorphism of  $\mathbb{R}^n$ ,  $n \geq 3$ , such that every orbit of the homeomorphism is dense?* T. Homma and S. Kinoshita [136] had proved the following theorem in 1953, which provides a negative answer to both Problem 1084 and Problem 1085: If  $X$  is a locally compact, noncompact, separable metric space then for any continuous self-map of  $X$  the set of all points with a dense orbit has empty interior in  $X$ . N.C. Bernardes Jr. [36] proved a generalization of this theorem to locally compact spaces which are not necessarily metrizable. Bernardes proved that if  $X$  is a locally compact Hausdorff space which is not compact and has no isolated points, then for every continuous self map of  $X$ , the set of all points with a dense orbit has empty interior in  $X$ .

**Problem 1086.** *Characterize the planar continua admitting expansive homeomorphisms.* There are several results related to this problem. H. Kato proved that following classes of continua do not admit expansive homeomorphisms: planar Peano continua [148]; decomposable circle-like continua [152]; arc-like continua [150]. Kato [151] proved that planar continua admitting expansive homeomorphisms contain indecomposable continua. Kato [149] proved that if the shift homeomorphism of an inverse limit  $X$  of a graph with one bonding map is expansive and  $X$  can be embedded to the plane, then  $X$  separates the plane into  $n$  components ( $n > 3$ ). C. Mouron [206] proved that tree-like continua (hence all one-dimensional nonseparating plane continua) do not admit expansive homeomorphisms. Mouron proved that if  $X$  is a one-dimensional plane continuum that separates the plane into two pieces, then  $X$  cannot admit an expansive homeomorphism. The Plykin attractor

1	2	3	4	5	6	7	8	9	10	11	12	13
14	15	16	17	18	19	20	21	22	23	24	25	26
27	28	29	30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49	50	51	52
53	54	55	56	57	58	59	60	61	62	63	64	65
66	67	68	69	70	71	72	73	74	75	76	77	78
79	80	81	82	83	84	85	86	87	88	89	90	91
92	93	94	95	96	97	98	99	100	101	102	103	104
105	106	107	108	109	110	111	112	113	114	115	116	117
118	119	120	121	122	123	124	125	126	127	128	129	130
131	132	133	134	135	136	137	138	139	140	141	142	143
144	145	146	147	148	149	150	151	152	153	154	155	156
157	158	159	160	161	162	163	164	165	166	167	168	169
170	171	172	173	174	175	176	177	178	179	180	181	182
183	184	185	186	187	188	189	190	191	192	193	194	195
196	197	198	199	200	201	202	203	204	205	206	207	208
209	210	211	212	213	214	215	216	217	218	219	220	221
222	223	224	225	226	227	228	229	230	231	232	233	234
235	236	237	238	239	240	241	242	243	244	245	246	247
248	249	250	251	252	253	254	255	256	257	258	259	260
261	262	263	264	265	266	267	268	269	270	271	272	273
274	275	276	277	278	279	280	281	282	283	284	285	286
287	288	289	290	291	292	293	294	295	296	297	298	299
300	301	302	303	304	305	306	307	308	309	310	311	312
313	314	315	316	317	318	319	320	321	322	323	324	325
326	327	328	329	330	331	332	333	334	335	336	337	338
339	340	341	342	343	344	345	346	347	348	349	350	351
352	353	354	355	356	357	358	359	360	361	362	363	364
365	366	367	368	369	370	371	372	373	374	375	376	377
378	379	380	381	382	383	384	385	386	387	388	389	390
391	392	393	394	395	396	397	398	399	400	401	402	403
404	405	406	407	408	409	410	411	412	413	414	415	416
417	418	419	420	421	422	423	424	425	426	427	428	429
430	431	432	433	434	435	436	437	438	439	440	441	442
443	444	445	446	447	448	449	450	451	452	453	454	455
456	457	458	459	460	461	462	463	464	465	466	467	468
469	470	471	472	473	474	475	476	477	478	479	480	481
482	483	484	485	486	487	488	489	490	491	492	493	494
495	496	497	498	499	500	501	502	503	504	505	506	507
508	509	510	511	512	513	514	515	516	517	518	519	520
521	522	523	524	525	526	527	528	529	530	531	532	533
534	535	536	537	538	539	540	541	542	543	544	545	546
547	548	549	550	551	552	553	554	555	556	557	558	559

Fig. 1. Status of Problems 1–559.

560	561	562	563	564	565	566	567	568	569	570	571	572
573	574	575	576	577	578	579	580	581	582	583	584	585
586	587	588	589	590	591	592	593	594	595	596	597	598
599	600	601	602	603	604	605	606	607	608	609	610	611
612	613	614	615	616	617	618	619	620	621	622	623	624
625	626	627	628	629	630	631	632	633	634	635	636	637
638	639	640	641	642	643	644	645	646	647	648	649	650
651	652	653	654	655	656	657	658	659	660	661	662	663
664	665	666	667	668	669	670	671	672	673	674	675	676
677	678	679	680	681	682	683	684	685	686	687	688	689
690	691	692	693	694	695	696	697	698	699	700	701	702
703	704	705	706	707	708	709	710	711	712	713	714	715
716	717	718	719	720	721	722	723	724	725	726	727	728
729	730	731	732	733	734	735	736	737	738	739	740	741
742	743	744	745	746	747	748	749	750	751	752	753	754
755	756	757	758	759	760	761	762	763	764	765	766	767
768	769	770	771	772	773	774	775	776	777	778	779	780
781	782	783	784	785	786	787	788	789	790	791	792	793
794	795	796	797	798	799	800	801	802	803	804	805	806
807	808	809	810	811	812	813	814	815	816	817	818	819
820	821	822	823	824	825	826	827	828	829	830	831	832
833	834	835	836	837	838	839	840	841	842	843	844	845
846	847	848	849	850	851	852	853	854	855	856	857	858
859	860	861	862	863	864	865	866	867	868	869	870	871
872	873	874	875	876	877	878	879	880	881	882	883	884
885	886	887	888	889	890	891	892	893	894	895	896	897
898	899	900	901	902	903	904	905	906	907	908	909	910
911	912	913	914	915	916	917	918	919	920	921	922	923
924	925	926	927	928	929	930	931	932	933	934	935	936
937	938	939	940	941	942	943	944	945	946	947	948	949
950	951	952	953	954	955	956	957	958	959	960	961	962
963	964	965	966	967	968	969	970	971	972	973	974	975
976	977	978	979	980	981	982	983	984	985	986	987	988
989	990	991	992	993	994	995	996	997	998	999	1000	1001
1002	1003	1004	1005	1006	1007	1008	1009	1010	1011	1012	1013	1014
1015	1016	1017	1018	1019	1020	1021	1022	1023	1024	1025	1026	1027
1028	1029	1030	1031	1032	1033	1034	1035	1036	1037	1038	1039	1040
1041	1042	1043	1044	1045	1046	1047	1048	1049	1050	1051	1052	1053
1054	1055	1056	1057	1058	1059	1060	1061	1062	1063	1064	1065	1066
1067	1068	1069	1070	1071	1072	1073	1074	1075	1076	1077	1078	1079
1080	1081	1082	1083	1084	1085	1086	1087	1088	1089	1090	1091	1092
1093	1094	1095	1096	1097	1098	1099	1100					

Fig. 2. Status of Problems 560–1100.

is a one-dimensional plane continuum that separates the plane into four pieces and does admit an expansive homeomorphism. It is not known whether a one-dimensional planar continuum that separates the plane into three pieces can admit an expansive homeomorphism. Mourn gave an example of a two-dimensional plane continuum (which separates the plane into seven pieces) that admits an expansive homeomorphism. It is still unknown if a two-dimensional nonseparating plane continuum admits an expansive homeomorphism.

### One-dimensional versus two-dimensional dynamics by S. van Strien

There have been no solutions to the thirteen problems in this section.

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